

Corrigé Type

Systèmes mécaniques articulés et robotique: Session normale (2019/2020)

1^{ère} Master Construction mécanique

Solution :

(a) $H_2^0 = H_{rot,z}(30^\circ)H_1^0$

$$= \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ & 0 & 0 \\ \sin 30^\circ & \cos 30^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2 pts

$$= \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ & 0 & 3\cos 30^\circ - 5\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ & 0 & 3\sin 30^\circ + 5\cos 30^\circ \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.866 & -0.500 & 0 & 0.098 \\ 0.500 & 0.866 & 0 & 5.830 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b)

$$H_2^0 = H_{rot,x}(-25^\circ)H_1^0$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-25^\circ) & -\sin(-25^\circ) & 0 \\ 0 & \sin(-25^\circ) & \cos(-25^\circ) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3 pts

$$= \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0.906 & 0.422 & 7.489 \\ 0 & -0.422 & 0.906 & 4.231 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(c)

$$\begin{aligned}
H_2^0 &= H_1^0 H_{rot,y}(40^\circ) \\
&= \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 40^\circ & 0 & \sin 40^\circ & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 40^\circ & 0 & \cos 40^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} \cos 40^\circ & 0 & \sin 40^\circ & 3 \\ 0 & 1 & 0 & 5 \\ -\sin 40^\circ & 0 & \cos 40^\circ & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.766 & 0 & 0.642 & 3 \\ 0 & 1 & 0 & 5 \\ -0.642 & 0 & 0.766 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}$$

3 pts

(d)

$$H_2^0 = H_{trans}(0, 8, 0) H_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 13 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3 pts

(e)

$$\begin{aligned}
H_3^0 &= H_{trans}(0, 3, 0) H_{rot,x}(30^\circ) H_1^0 \\
&= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ & 0 \\ 0 & \sin 30^\circ & \cos 30^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0.866 & -0.5 & 3.83 \\ 0 & 0.5 & 0.866 & 8.562 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}$$

3 pts

(f)

$$\begin{aligned}
H_1^0 &= H_{rot,y}(30^\circ) H_1^0 H_{trans}(0, 0, 5) \\
&= \begin{bmatrix} \cos 30^\circ & 0 & \sin 30^\circ & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 30^\circ & 0 & \cos 30^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.866 & 0 & 0.5 & 8.598 \\ 0 & 1 & 0 & 5 \\ -0.5 & 0 & 0.866 & 8.892 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}$$

3 pts

(g)

$$\begin{aligned}
H_1^0 &= H_{trans}(-2, 0, 0) H_1^0 H_{rot,y}(15^\circ) \\
&= \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 15^\circ & 0 & \sin 15^\circ & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 15^\circ & 0 & \cos 15^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.965 & 0 & 0.258 & 1 \\ 0 & 1 & 0 & 5 \\ -0.258 & 0 & 0.965 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}$$

3 pts