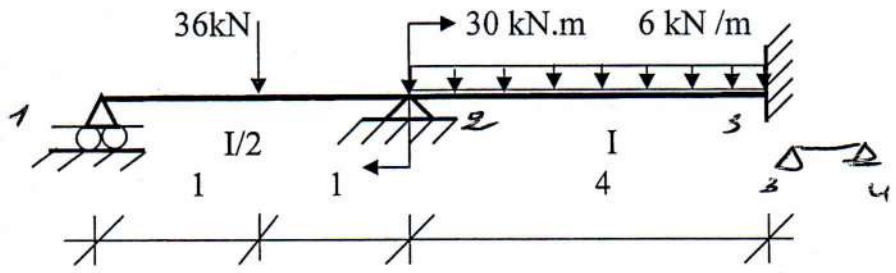
correction Sujet I

Exo 1



appuis 1 2 3

$$\frac{2}{\frac{1}{2}} M_1 + 2\left(\frac{2}{\frac{1}{2}} + \frac{4}{1}\right) M_2 + \frac{4}{1} M_3 = -\frac{6}{2\times\frac{1}{2}} H_1 - \frac{6}{4\times 1} H_3$$

$$16 M_2 + 4 M_3 = -6 H_1 - \frac{3}{2} H_3$$

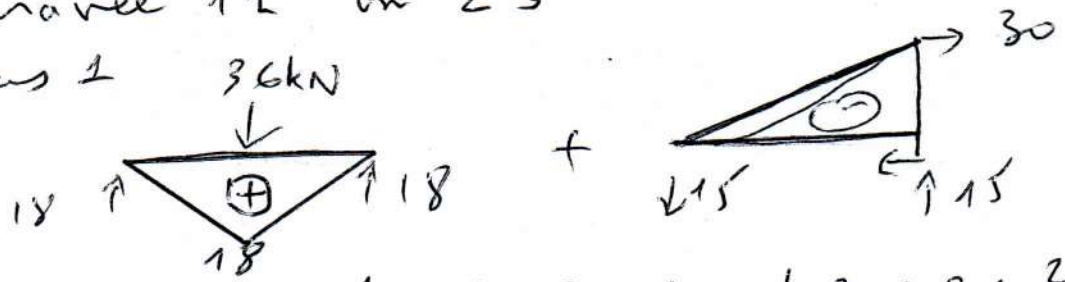
appuis 2 3 4

$$4 M_2 + 2(4+0) M_3 = -\frac{6}{4} H_2$$

$$4 M_2 + 8 M_3 = -\frac{3}{2} H_2$$

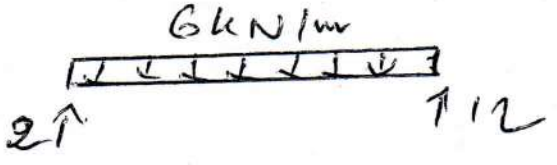
Le moment 30 peut être appliqué soit sur la travée 1 2 ou 2 3

cas 1



$$H_1 = \frac{1}{2} 18 \times 2 \times 1 - \frac{1}{2} 30 \times 2 \times \frac{2}{3} \times 2 =$$

$$18 - 40 = \boxed{-22} \quad (0,5)$$

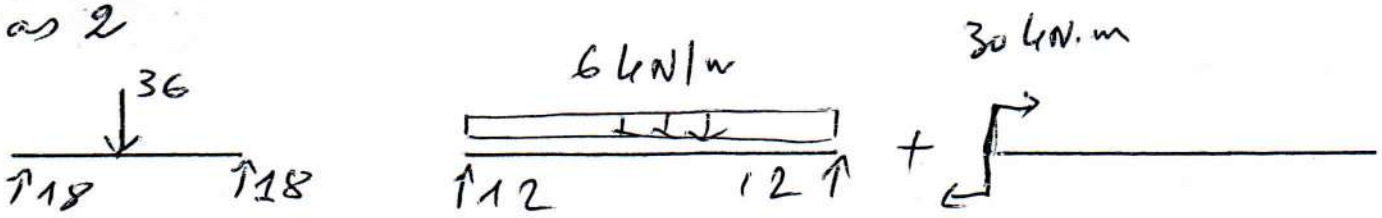


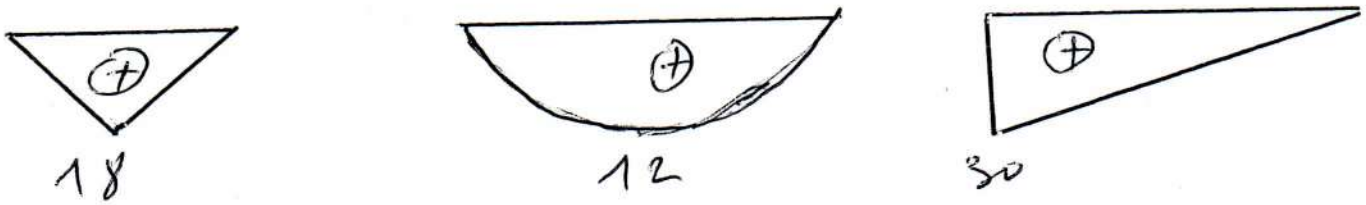
$$H_2 = H_3 = \frac{2}{3} 12 \times 4 \times 2 = \boxed{64}$$

$2 \times (0,5)$

$$\frac{q l^2}{8} = 12$$

Cas 2





$H_1 = \frac{1}{2} \cdot 18 \cdot 2 \cdot 1 = \boxed{18}$ (0,5)

$H_2 = \frac{2}{3} \cdot 12 \cdot 4 \cdot 2 + \frac{1}{2} 30 \cdot 4 \times \frac{1}{3} \cdot 4 = \boxed{144}$ (0,5)

$H_3 = \quad " \quad + \frac{1}{2} 30 \cdot 4 \times \frac{2}{3} \cdot 4 = \boxed{24}$ (0,5)

Cas 1 (2 x 1)

$$\begin{bmatrix} 16 & 4 \\ 4 & 8 \end{bmatrix} \begin{Bmatrix} M_2 \\ M_3 \end{Bmatrix} = \begin{Bmatrix} 36 \\ -96 \end{Bmatrix}$$

ou

$$\begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} M_2 \\ M_3 \end{Bmatrix} = \begin{Bmatrix} 9 \\ -24 \end{Bmatrix}$$

$\Delta = 8 - 1 = 7$

$\Delta_{M_2} = 9 \times 2 + 24 \times 1 = 42$

$M_2 = \frac{42}{7} = 6 \text{ kN.m}$

$\Delta_{M_3} = -4 \times 24 + 1 \times 9 = -105$

$M_3 = -\frac{105}{7} = -15 \text{ kN.m}$

2 x 0,5

Cas 2 (2 x 1)

$$\begin{bmatrix} 16 & 4 \\ 4 & 8 \end{bmatrix} \begin{Bmatrix} M_2 \\ M_3 \end{Bmatrix} = \begin{Bmatrix} 144 \\ -216 \end{Bmatrix}$$

ou

$$\begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} M_2 \\ M_3 \end{Bmatrix} = \begin{Bmatrix} -111 \\ -54 \end{Bmatrix}$$

$\Delta = 8 - 1 = 7$

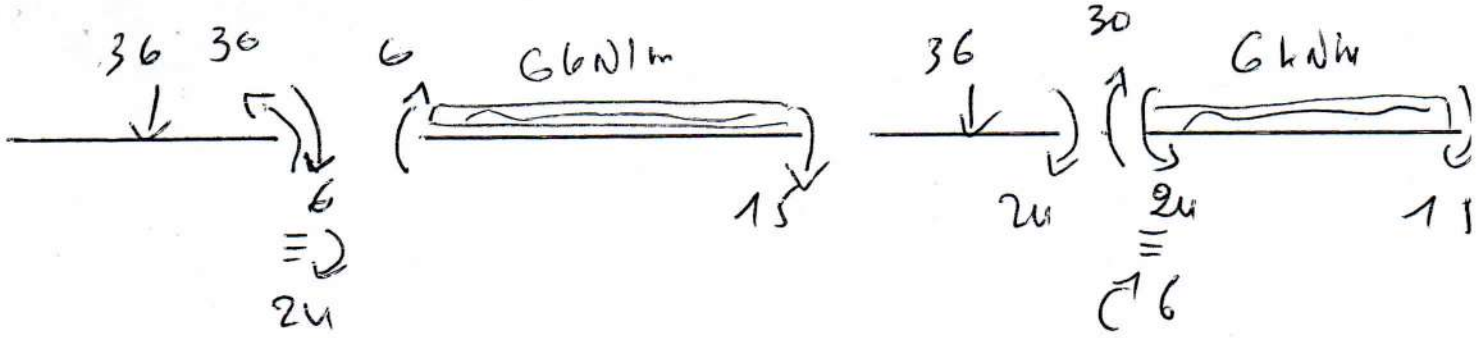
$\Delta_{M_2} = -111 \times 2 + 54 = -168$

$M_2 = -\frac{168}{7} = -24 \text{ kN.m}$

$\Delta_{M_3} = -4 \times 54 + 111 = -105$

$M_3 = -\frac{105}{7} = -15 \text{ kN.m}$

2 x 0,5



tranee 1 2

$$M = \mu + \overset{0}{M_1} + \frac{M_2 - \overset{0}{M_1}}{2} x = \mu + \frac{M_2}{2} x$$

Section I - I $0 \leq x \leq 1$

$$\mu = 18x$$

(2) $$M = 18x - \frac{24}{2} x = 6x \begin{cases} x=0, \ M=0 \\ x=1, \ M=6\ kN.m \end{cases}$$

$$T = \frac{\partial M}{\partial x} = 6\ kN$$

Section II - II $1 \leq x \leq 2$

$$M = 6x - 36(x-1) = -30x + 36$$

(2) $x = 1, \ M = 6\ kN.m$

$x = 2, \ M = -24\ kN.m$

$$T = \frac{\partial M}{\partial x} = -30\ kN$$

tranee 2-3

$$\mu = 12x - 3x^2$$

$$M = \mu + M_2 + \frac{M_3 - M_2}{4} x =$$

(2) $$12x - 3x^2 + 6 + \frac{-15-6}{4} x = -3x^2 + 6{,}75x + 6$$

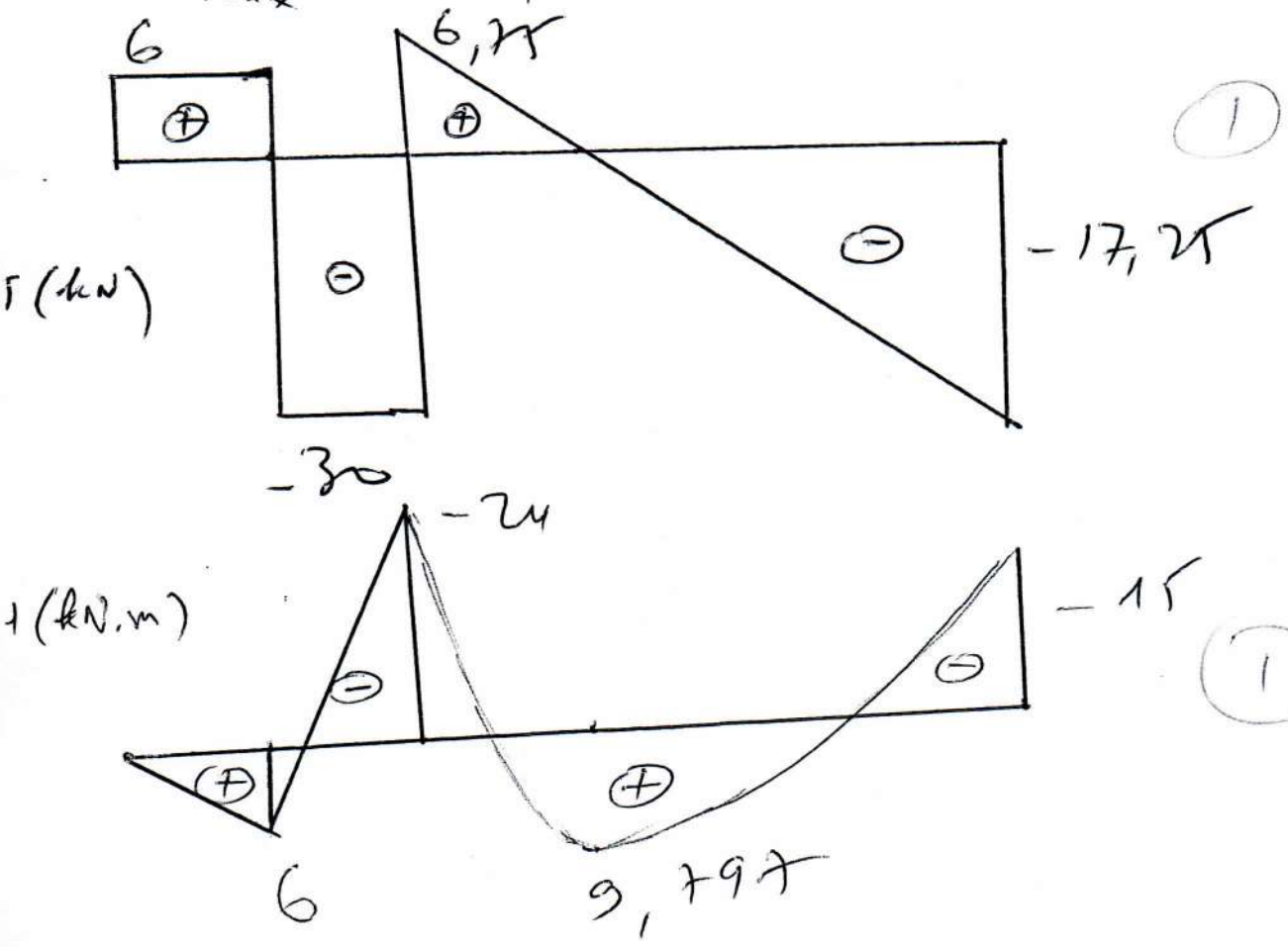
$x = 0, \ M = 6\ kN.m$

$x = 4, \ M = -15\ kN.m$

$$T = \frac{\partial M}{\partial x} = -6x + 6{,}75 \begin{cases} x=0, \ T=6{,}75\ kN \\ x=4, \ T=-17{,}25\ kN.m \end{cases}$$

$T = 0 \Rightarrow x = \frac{6,25}{6} = 1,125$

$M_{max} = 9,797\ kN.m$



(1)

(1)

$R_1 = 6\ kN$

$R_2 = 36,75\ kN$

$R_3 = 17,25$

$M = 15\ kN$

3 x 0,5

Exo 2

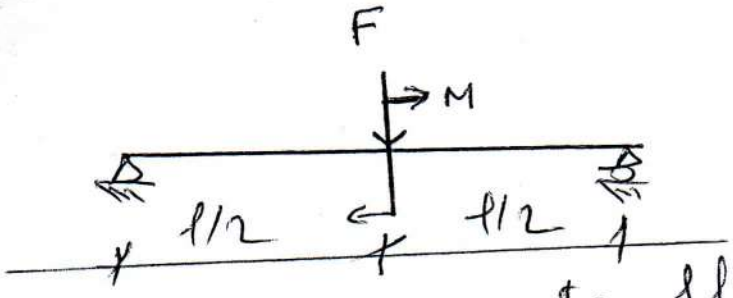
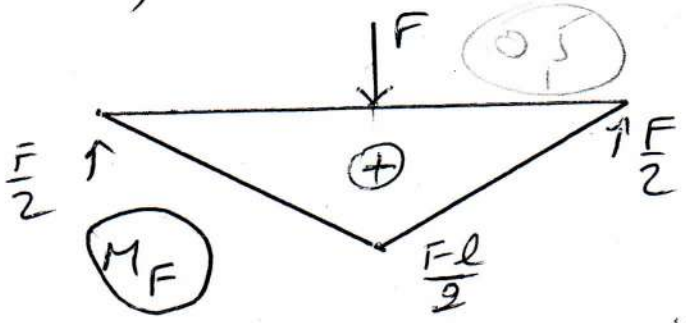
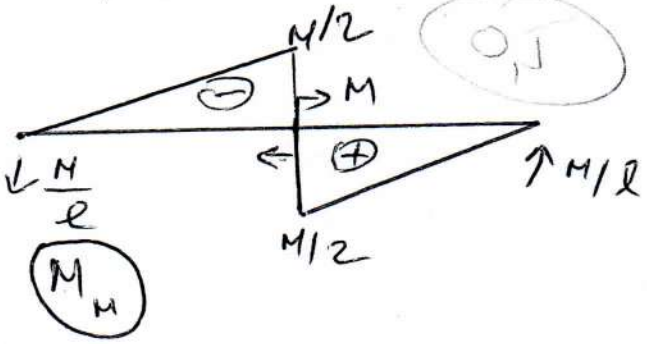


diagramme des moments fléchissants



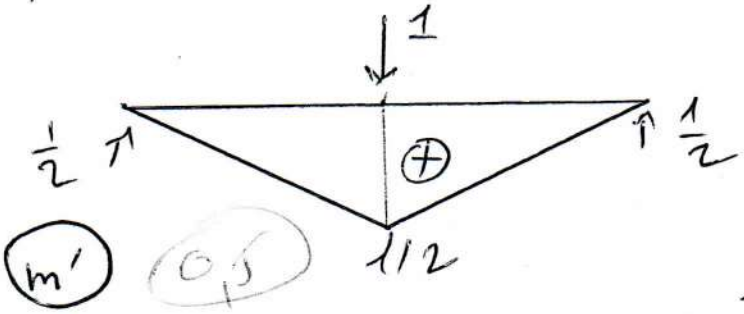
0,5



0,5

On utilise la methode de Maxwell-Mohr

flèche au milieu



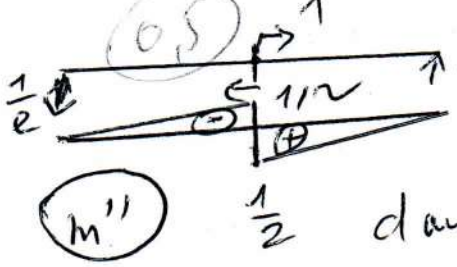
0,5

$$EI\Delta = \int_l M_F m' ds + \int_l M_M m' ds$$

2

le 2ème terme est nul car m' est symétrique et M_M est antisymetrique donc Δ n'est dû qu'à F

rotation au milieu



0,5

$$EI\theta = \int_l M_F m'' ds + \int_l M_M m'' ds$$

dans ce cas le 1er terme est nul car M_F est symétrique et m'' est antisymetrique (leur produit = 0) donc la rotation n'est dû à M

2