* Construire l'AEF qui reconnait le langage $L = \{(ab+b)^* ba\}$.

* $L\|a = (ab+b)^*\|a \cdot ba + \underbrace{f((ab+b)^*)}_{\varepsilon} \cdot \underbrace{(ba)\|a}_{\phi} =$

$(ab+b)\|a \cdot (ab+b)^* \cdot ba = (\underbrace{ab\|a}_{b} + \underbrace{b\|a}_{\phi}) \cdot (ab+b)^* \cdot ba = \underbrace{\boxed{b \cdot (ab+b)^* \cdot ba}}_{q_1}$

* $L\|b = (ab+b)^*\|b \cdot ba + \underbrace{\underbrace{f((ab+b)^*)}_{\varepsilon} \cdot \underbrace{(ba)\|b}_{a}}_{a} =$

$(\underbrace{ab\|b}_{\phi} + \underbrace{b\|b}_{\varepsilon}) \cdot (ab+b)^* \cdot ba + a = \boxed{(ab+b)^* ba + a}$ q_2

* $q_1\|a = (b \cdot (ab+b)^* \cdot ba)\|a = \underbrace{b\|a}_{\phi} \cdot (ab+b)^* ba + \underbrace{\underbrace{f(b)}_{\phi} \cdot ((ab+b)^* ba)_a}_{\phi}$

$= \phi$ (Donc pas de transition avec le $\boxed{a}$).

* $q_1\|b = \underbrace{b\|b}_{\varepsilon} \cdot (ab+b)^* ba + \underbrace{\underbrace{f(b)}_{\phi} \cdot ((ab+b)^* ba)}_{\phi} = \underbrace{\boxed{(ab+b)^* ba}}_{q_0}$

- $q_2 \| a = \left((ab+b)^* ba + a\right)\|a = \left((ab+b)^* ba\right)\|a + \underbrace{a\|a}_{\varepsilon} =$

$(ab+b)^*\|a \cdot ba + \underbrace{f\left((ab+b)^*\right) \cdot \underbrace{(ba)\|a}_{\phi}}_{\phi} + \varepsilon = (\underbrace{ab\|a}_{b} + \underbrace{b\|a}_{\phi}) \cdot (ab+b)^* \cdot ba +$

$= \boxed{b \cdot (ab+b)^* \cdot ba + \varepsilon}$ q_3

- $q_2 \| b = \left((ab+b)^* ba\right)\|b + \underbrace{a\|b}_{\phi} =$

$(ab+b)^*\|b \cdot ba + \underbrace{f\left((ab+b)^*\right)}_{\varepsilon} \cdot \underbrace{(ba\|b)}_{a} =$

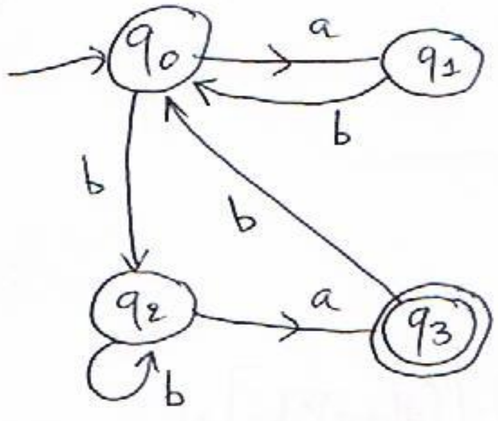
$(\underbrace{ab\|a}_{b} + \underbrace{b\|a}_{\phi}) \cdot (ab+b)^* \cdot ba + a = \boxed{b(ab+b)^* \cdot ba + a}$ q_2

- $q_3 \| a = \left(b(ab+b)^* ba + \varepsilon\right)\|a = \left(b(ab+b)^* ba\right)\|a + \phi =$

$\underbrace{b\|a}_{\phi} \cdot (ab+b^*) ba + \underbrace{f(b)}_{\phi} \cdot \left((ab+b)^* ba\right)\|a = \phi$ (Pas de transiti

- $q_3 \| b = \underbrace{b\|b}_{\varepsilon} \cdot (ab+b^*) ba + \underbrace{\underbrace{f(b)}_{\phi} \cdot \left((ab+b)^* ba\right)\|b}_{\phi} = \boxed{(ab+b)^* ba}$ q_0

D'où, l'AEF résultant est le suivant :



Construire l'AEF qui reconnait le langage $L = \{(ab+a)^* ba\}$.

- $L_{\|a} = (ab+a)^*_{\|a} \cdot ba + \underbrace{f((ab+a)^*)}_{\varepsilon} \cdot \underbrace{(ba)_{\|a}}_{\phi} =$

$(\underbrace{ab_{\|a}}_{b} + \underbrace{a_{\|a}}_{\varepsilon}) \cdot (ab+a)^* \cdot ba = \boxed{(b+\varepsilon) \cdot (ab+a)^* \cdot ba}$ q_1

- $L_{\|b} = (\underbrace{ab_{\|b}}_{\phi} + \underbrace{a_{\|b}}_{\phi}) \cdot (ab+a)^* \cdot ba + \underbrace{f((ab+a)^*)}_{\varepsilon} \cdot \underbrace{(ba)_{\|b}}_{a} = \boxed{a}$ q_2

- $q_{2\|b} = \phi$. $q_{2\|a} = \boxed{\varepsilon}$ q_3.

$q_{3\|a} = q_{3\|b} = \phi$.

$$* \; q_1 \| a = \left[(b+\varepsilon)\cdot(ab+a)^*.ba \right]_{\|a} = \overbrace{(b+\varepsilon)_{\|a}}^{\phi}\cdot(ab+a)^*.ba + \underbrace{f(b+\varepsilon)}_{\varepsilon}\cdot\left[(ab+a)^*.ba\right]_{\|a}$$

$$= \left[(ab+a)^*.ba\right]_{\|a} = L_{\|a} = q_1$$

$$* \; q_1 \| b = \underbrace{(b+\varepsilon)_{\|b}}_{\varepsilon}\cdot(ab+a)^*.ba + \underbrace{f(b+\varepsilon)}_{\varepsilon}\cdot\left[(ab+a)^*.ba\right]_{\|b}$$

$$= (ab+a)^*.ba + L_{\|b} = \boxed{(ab+a)^*.ba+a} \; q_4$$

$$* \; q_4 \| a = \left[(ab+a)^*.ba\right]_{\|a} + a\|a = L_{\|a} + \varepsilon =$$

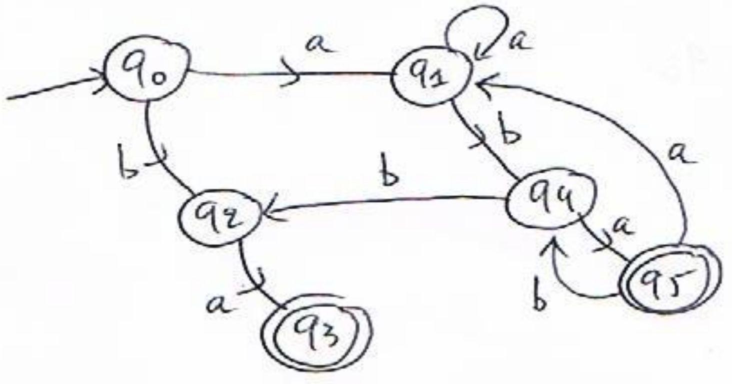
$$\boxed{(b+\varepsilon)\cdot(ab+a)^*.ba+\varepsilon} \; q_5 \approx q_1+\varepsilon$$

$$* \; q_4 \| b = L_{\|b} + a\|b = L_{\|b} = q_2 .$$

$$q_5\|a = q_1\|a = q_1$$

$$q_5\|b = q_1\|b = q_4$$

AEF résultant :



Solution de l'exercice n°07 (Série des exos supp).

$$\begin{cases} X_0 = aX_1 \\ X_1 = bX_1 + aX_2 \\ X_2 = aX_2 + bX_1 + \varepsilon \end{cases} \Rightarrow \begin{cases} X_2 = a^* \cdot (bX_1 + \varepsilon) = \boxed{a^*bX_1 + a^*} \\ X_1 = bX_1 + aX_2 = bX_1 + a^+bX_1 + a^+ \\ \quad = (b + a^+b)X_1 + a^+ \end{cases}$$

$$\Rightarrow \boxed{X_1 = (b + a^+b)^* . a^+}$$

$$\Rightarrow X_0 = aX_1 = \boxed{a(b + a^+b)^* . a^+} = \text{Langage reconnu.}$$

Exercice n°08 :

$$\begin{cases} X_0 = bX_0 + aX_1 + \varepsilon \\ X_1 = aX_1 + bX_2 + \varepsilon \\ X_2 = bX_0 \end{cases} \Rightarrow \begin{cases} X_0 = b^*(aX_1 + \varepsilon) = b^*aX_1 + b^* \\ X_1 = a^*(bX_2 + \varepsilon) = a^*bX_2 + a^* \\ \boxed{X_1 = a^*bbX_0 + a^*} \end{cases}$$

$$X_0 = b^*aa^*bbX_0 + b^*aa^* + b^* = (b^*a^+b^2)X_0 + b^*a^+ + b^*$$

$$\Rightarrow \boxed{X_0 = (b^*a^+b^2)^* . [b^* + b^*a^+]} = \text{Langage reconnu}$$