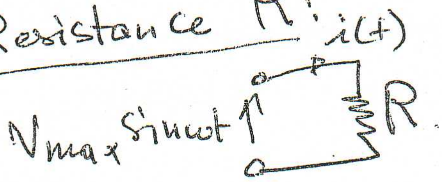
Correction : Examen électrotechnique

2ème année – électrotechnique fondamentale

1/ Question de cours

1) Cas d'une Resistance R :



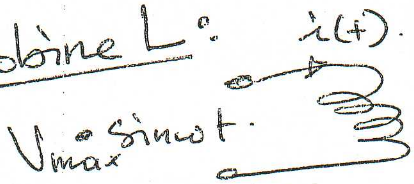
$$i(t) = \frac{V_{max}}{R} \cdot \sin\omega t \Rightarrow p(t) = \frac{V_{max}^2}{R} \sin^2\omega t$$

$$\sin^2\omega t = \frac{1}{2}(1 - \cos 2\omega t)$$

$$\Rightarrow P_{moy} = \frac{1}{T}\int_0^T p(t)\,dt = \frac{V_{max}^2}{2RT}\int_0^T (1 - \cos 2\omega t)\,dt$$

$$\Rightarrow P_{moy} = \frac{V_{max}^2}{2R} = V_{eff} \cdot I_{eff} \longrightarrow \text{(3 pts)}$$

2/ Cas d'une bobine L :



$$V_L(t) = L\frac{di}{dt}$$

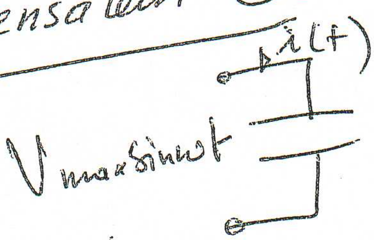
$$\Rightarrow i(t) = \frac{V_{max}}{L}\int \sin\omega t\,dt = -\frac{V_{max}}{L\omega} \cdot \cos\omega t$$

$$\Rightarrow p(t) = -\frac{V_{max}^2}{L\omega} \cdot \sin\omega t \cdot \cos\omega t$$

$$\sin\omega t \cdot \cos\omega t = \frac{1}{2}\sin(2\omega t)$$

$$\Rightarrow P_{moy} = -\frac{V_{max}^2}{2TL\omega}\int_0^T \sin 2\omega t\,dt = 0 \text{ — (1 pt)}$$

3 - Cas d'un Condensateur C :



$$i(t) = C\frac{dv}{dt}$$

$$i(t) = C\omega V_{max} \cos\omega t$$

$$p(t) = C\omega V_{max}^2 \cdot \sin\omega t \cdot \cos\omega t = \frac{C\omega V_{max}^2}{2}\sin(2\omega t)$$

$$\Rightarrow P_{moy} = \frac{C\omega V_{max}^2}{2T}\int_0^T \sin 2\omega t\,dt = 0 \quad \text{(1 pt)}$$

Solution Exercice 1

1/. $F_{moy} = \frac{1}{T}\int_0^T f(t)\,dt$ —— (1 pt)

2) $F_{eff} = \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt}$ —— (1 pt)

(3)- La période du signal d'entrée $v(t)$ et $T_1 = 20ms$

donc : $V_1(t) = \begin{cases} V_m \cdot \sin\omega t & 0 \leq t \leq T_1/2 \\ 0 & \frac{T_1}{2} \leq t \leq T_1 \end{cases}$

- La Valeur Moyenne de $v_1(t)$:

$$V_{1Moy} = \frac{1}{T_1}\int_0^{T_1/2} V_m \sin\omega t\,dt = \frac{V_m}{T_1\omega} \times 2 =$$

$$\omega = \frac{2\pi}{T_1} \Rightarrow \boxed{V_{1moy} = \frac{V_m}{\pi}}$$ (1,25 pts)

- La Valeur efficace de $V_1(t)$

$$V_{1eff}^2 = \frac{1}{T_1}\int_0^{T_1/2} V_m^2 \cdot \sin^2\omega t\,dt$$

On sait que : $\sin^2\omega t = \frac{1}{2}(1 - \cos 2\omega t)$

$$\Rightarrow V_{1eff}^2 = \frac{V_m^2}{T_1 2} \cdot \frac{T_1}{2} = \frac{V_m^2}{4}$$

$$\Rightarrow \boxed{V_{1eff} = \frac{V_m}{2}}$$ → (1,25 pts)

(4)- $V_2(t)$ est un signal periodique de periode $T_2 = 10ms$

$$T_2 = \frac{T_1}{2} \Rightarrow V_{2moy} = \frac{1}{T_2}\int_0^{T_2} V_m \cdot \sin\omega t\,dt$$

$$= \frac{V_m}{\ldots} \cdot 2 = \frac{2V_{max}}{\pi}$$ (1,25)

- La valeur efficace de $V_2(t)$:

$$V_{2eff}^2 = \frac{1}{T_2}\int_0^{T_2} V_m^2 \sin^2\omega t \, dt$$

$$\Rightarrow \boxed{V_{eff} = \frac{V_m}{\sqrt{2}}}$$ (1,25 pt)

Solution exercice N°2 :

$T = 8\text{ div.} \times 2{,}5\text{ ms} = 20\text{ ms}$, $f = 50\text{ Hz}$, $\omega = 100\pi\text{ rad}$

1/ $i(t) = I_m \cdot \sin\omega t$

Voie A $\rightarrow U_L(t)$, Voie B $\rightarrow U_R(t)$.

- On : $U_R(t) = Ri(t) \rightarrow$ en phase avec $i(t)$.

$U_R(t) = 10 \sin\omega t$ ⟶ (1 pt)

- d'après les courbe $U_B(t)$ est en avance dephase.

$\Delta t = 2 \cdot 2{,}5 = 5\text{ ms} \rightarrow \varphi = \Delta t \cdot \omega = 90°$.

$\Rightarrow U_B(t) = 10 \sin(\omega t + 90°)$ ⟶ (1 pt)

2/- $U(t) = U_R(t) + U_L(t) \Rightarrow \underline{U} = \underline{U}_R + \underline{U}_L$ (phaseur).

$\underline{U}_R = 10 e^{J0}$, $\underline{U}_L = 10 e^{J90°} \Rightarrow \underline{U} = 10 e^{J0} + 10 e^{J90°}$

$\Rightarrow \underline{U} = 10 \cdot \sqrt{2}\, e^{J45°} \Rightarrow U(t) = 10\sqrt{2} \cdot \sin(\omega t + 45°)$ (1 pt)

3/ $|Z| = \frac{V_{eff}}{I_{eff}} = \frac{10}{0{,}0141} \cong 709\,\Omega$, $R = \frac{U_{Rmax}}{I_{max}} = 501{,}5\,\Omega$.

$|Z|^2 = R^2 + (L\omega)^2 \Rightarrow L = \sqrt{\frac{|Z|^2 - R^2}{\omega^2}}$

$$\boxed{L = 1{,}59\text{ H}}$$

4) $\text{tg}\phi = \frac{L\omega}{R} \cong 1 \Rightarrow \Phi \cong 45° \rightarrow$ cohérente avec la mesure. $(\varphi \cong \Delta t \cdot \omega \cong 45°)$. (2 pt)

(5) - $P_{act} = V_{eff} \cdot I_{eff} \cdot \cos\Phi = 0{,}1\,W$ — (0,5 pt)

- $Q = V_{eff}\, I_{eff} \sin\Phi = 0{,}1\,VAR$ — (0,5 pt)

- $S = U_{eff} I_{eff} = 0{,}141\,VA$ — (0,5 pt)

- $FP = \frac{P}{S} = \cos\Phi = 0{,}707$ → (0,5 pt)