Chapter 4: MIMO Systems

1. Introduction:

MIMO = Multiple Inputs Multiple Outputs

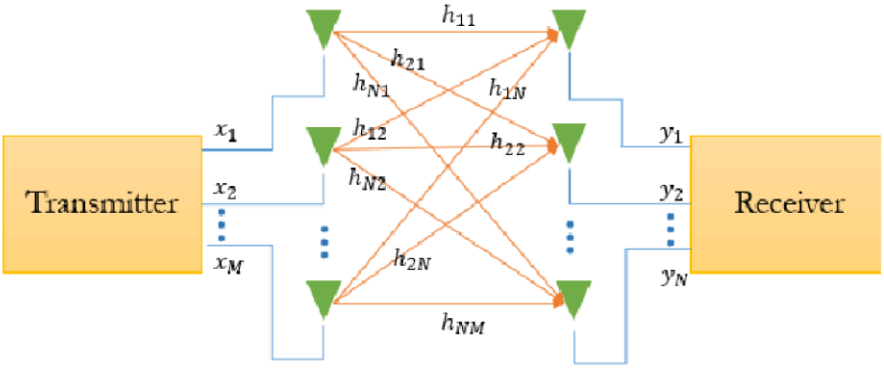
MIMO technology is used in 3G, 4G and 5G wireless communication systems.

The goal of MIMO technology is to increase the transmission rate.

Multiple inputs = multiple transmit antennas

Multiple outputs = multiple receiving antennas

2- Block diagram of MIMO system

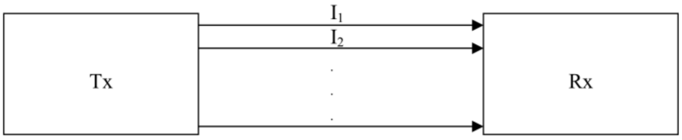


h_{ij} is the fading coefficient of the channel between each pair of antennas: transmission *i* and reception *j*

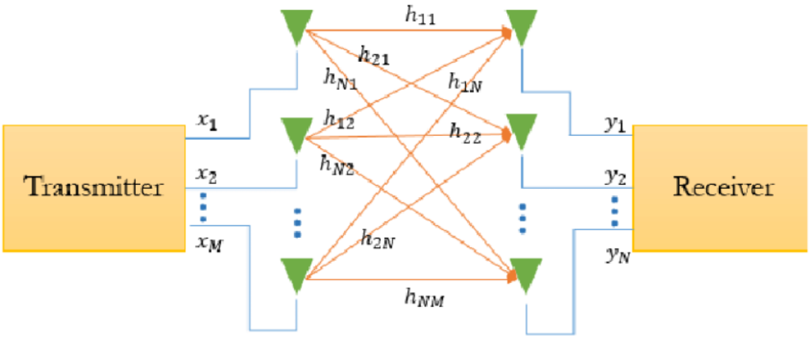
-Using multiple receiving antennas increases reliability through diversity.

3- Spatial Multiplexing

The advantage of the MIMO technique is not only reliability but also the increase in transmission rate, in MIMO systems we can transmit several streams of information in parallel, this property is called spatial multiplexing.



4- MIMO system model



$$X = \begin{bmatrix} x_1 \\ x_2 \\ . \\ x_M \end{bmatrix} \xrightarrow[\text{MIMO}]{\text{Canal}} Y = \begin{bmatrix} y_1 \\ y_2 \\ . \\ y_N \end{bmatrix}$$

$$Y = HX + W$$

Where *H* is the MIMO channel matrix and *W* is the noise.

$$H = \begin{bmatrix} h_{11}h_{12}\ldots\ldots h_{1M} \\ h_{21}h_{22}\ldots\ldots h_{2M} \\ .\quad .\quad \ldots\ldots\ldots\quad . \\ h_{N1}h_{N2}\ldots h_{NM} \end{bmatrix}, \; W = \begin{bmatrix} w_1 \\ w_2 \\ . \\ w_N \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ . \\ y_N \end{bmatrix} = \begin{bmatrix} h_{11}h_{12}\ldots\ldots h_{1M} \\ h_{21}h_{22}\ldots\ldots h_{2M} \\ .\quad .\quad \ldots\ldots\ldots\quad . \\ h_{N1}h_{N2}\ldots h_{NM} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ . \\ x_M \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ . \\ w_N \end{bmatrix}$$

In general, each element of the received vector *Y* can be written as follows:

$$y_j = h_{j1}x_1 + h_{j2}x_2 + \ldots + h_{jM}x_M$$

Special case of the MIMO system:

-If N=1, the system is called MISO

-If M=1, the system is called SIMO

-If M=N=1, the system is called SISO

5- MIMO receiver:

5-1 MIMO Zero forcing receiver (ZF receiver)

$$Y = HX + W$$

The problem is how to recover the transmitted vector *X* at the receiver?

Let the case be where $N \geq M$, in this case the number of receiving antennas is greater than the number of transmitting antennas.

In this case the number of equation *N* is greater than the number of unknowns *M*.

In order to determine *X*, we introduce the error:

$$e = Y - HX$$

to minimize the error *e* we minimize the square norm of *e*:

$$\min \|e\|^2 = \min \|Y - HX\|^2$$

Here we look for *X* which minimizes

$$\|Y - HX\|^2$$

(LS: Least Square problem)

To find the minimum, we look for the derivative of the error with respect to *X* and then set it to zero. *X* is a vector of dimension *M*.

Let *F(X)* is a function of *X*,

$$F(X) = \|Y - HX\|^2 == (Y - HX)^T (Y - HX) == (Y^T - X^T H^T)(Y - HX) = Y^T Y - Y^T HX - X^T H^T Y + X^T H^T HX$$

$$= Y^T Y - 2X^T H^T Y + X^T H^T HX$$

$$\frac{dF}{dX} = 0 - 2H^T Y + 2H^T HX = 0$$

$$H^T Y = H^T HX$$

$$\hat{X} = (H^T H)^{-1} H^T Y \text{, (zero forcing receiver)}$$

5-2 MIMO MMSE Receiver

MMSE: Minimum Mean Squared Error

The problem is to estimate X from given Y: $Y = \begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_N \end{bmatrix}$

We use a linear estimator of the form:

$$\hat{X} = C^T Y$$

The question is how we choose optimally the vector C^T?

We choose C^T that minimise the mean squared error $\left\|\hat{X} - X\right\|$

$\min E\left[\left\|\hat{X} - X\right\|^2\right]$

$\min E[\|C^T Y - X\|^2]$

$\|C^T Y - X\|^2 = (C^T Y - X)(C^T Y - X)^T$

$= (C^T Y - X)(Y^T C - X^T)$

$= C^T Y Y^T C - X Y^T C - C^T Y X^T + X X^T$

We note that:

$E[YY^T] = R_{yy}\ covariance\ matrix\ of\ Y$

$E[XY^T] = R_{xy}\ cross\ covariance\ matrix\ of\ X\ and\ Y$

$E[YX^T] = R_{yx} = R_{xy}^T$

So:

$\|C^T Y - X\|^2 = C^T R_{yy} C - R_{xy} C - C^T R_{yx} + R_{xx}$

$= C^T R_{yy} C - 2C^T R_{yx} + R_{xx} = f(c)$

$\min f(c) \rightarrow \dfrac{df}{dc} = 0$

$2R_{yy}C - 2R_{yx} = 0$

$C = R_{yy}^{-1} R_{yx}$ linear MMSE estimator

$$\hat{X} = C^T Y = R_{xy} R_{yy}^{-1} Y$$

Note: this development is for real vectors and we can generalize this to complex vectors and using instead of T transpose H Hermitian

$$\hat{X} = C^H Y = R_{xy} R_{yy}^{-1} Y$$

$$Y = HX + n$$

$$R_{yy} = E[YY^H] = E[(HX+n)(HX+n)^H] = E[(HX+n)(X^H H^H + n^H)]$$

$$= E[(HXX^H H^H + HXn^H + nX^H H^H + nn^H)]$$

To simplify this, we observe that the transmitted symbol X is uncorrelated with the noise n, so there is no correlation and $E[HXn^H] = E[nX^H H^H] = 0$

$$R_{yy} = E[(HXX^H H^H + nn^H)]$$

$$E[nn^H] = covariance\ of\ the\ noise = \sigma_n^2 = power\ of\ the\ noise$$

$$R_{yy} = HR_{XX}H^H + \sigma^2 I$$

$$R_{XX} = E[XX^H] = E\left[\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}\begin{bmatrix} x_1^* x_2^* \ldots & x_N^* \end{bmatrix}\right] = E\begin{bmatrix} |x_1^2| & x_1 x_2^* & .. \\ x_2 x_1^* & |x_2^2| & .. \\ .. & .. & |x_N^2| \end{bmatrix}$$

If the transmitted symbols are uncorrelated (independent symbols) we obtain:

$$R_{XX} = \begin{bmatrix} P & 0 & ..0 \\ 0 & p & ..0 \\ ..0 & ..0 & p \end{bmatrix} = PI$$

P is the power of the transmitted symbol

$$R_{yy} = PHH^H + \sigma_n^2 I = covariance\ matrix\ of\ the\ received\ symbol\ vector\ \text{Y}$$

We have

$$\hat{X} = C^H Y = R_{xy} R_{yy}^{-1} Y$$

Now we compute R_{xy}

$$R_{yx} = E[YX^H] = E[(HX+n)X^H] = E[HXX^H + nX^H] = PH$$

So we have

$$C^H = R_{xy} R_{yy}^{-1}$$

$$C = R_{yy}^{-1} R_{yx} = (PHH^H + \sigma_n^2 I)^{-1}.PH$$

$$\hat{X} = C^H Y = PH^H (PHH^H + \sigma_n^2 I)^{-1} Y$$ MIMO linear MMSE estimator (receiver)