Faculté de Technologie

Département de Télécommunication

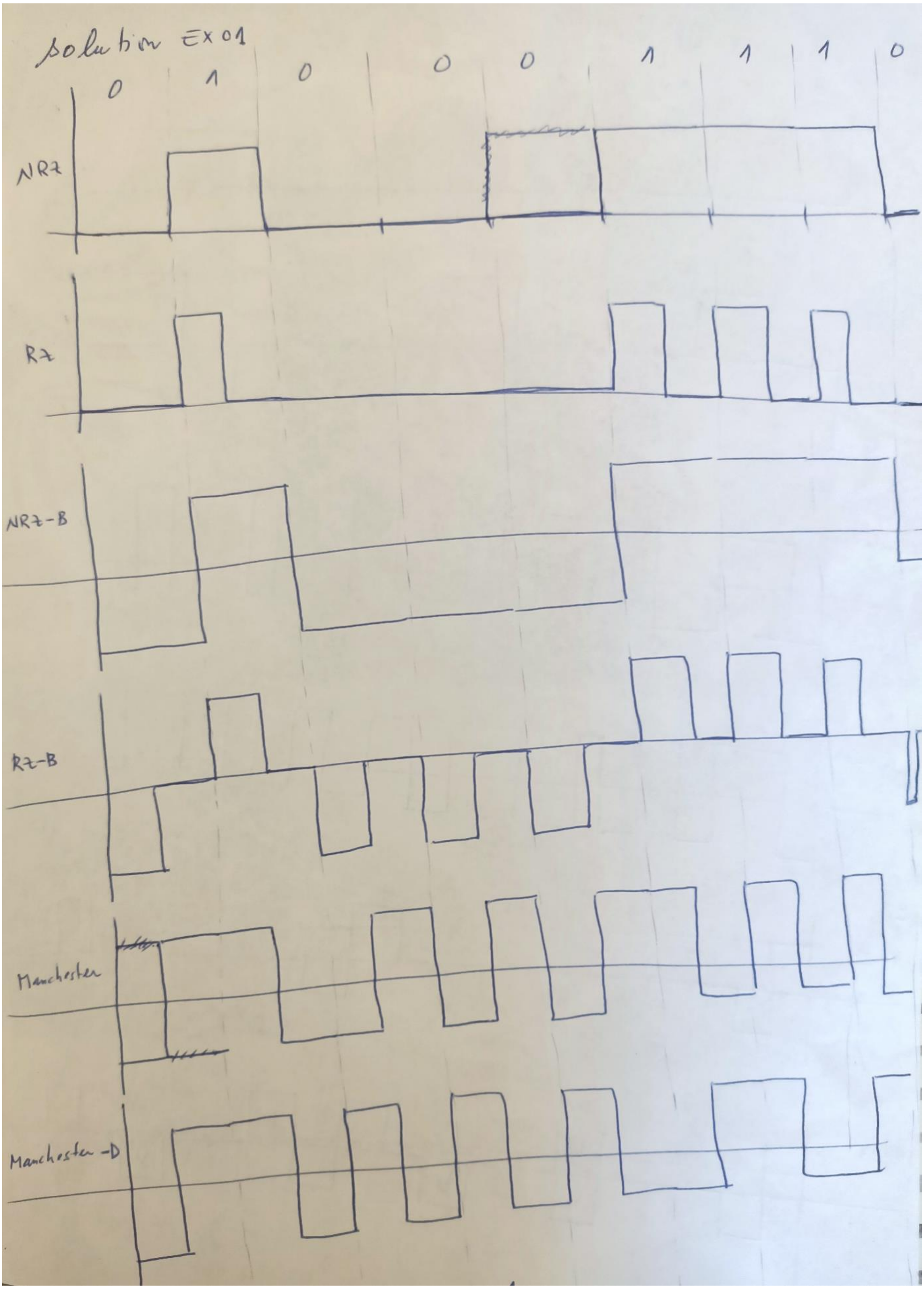
Filière : Télécommunication

Niveau : L3 TTL

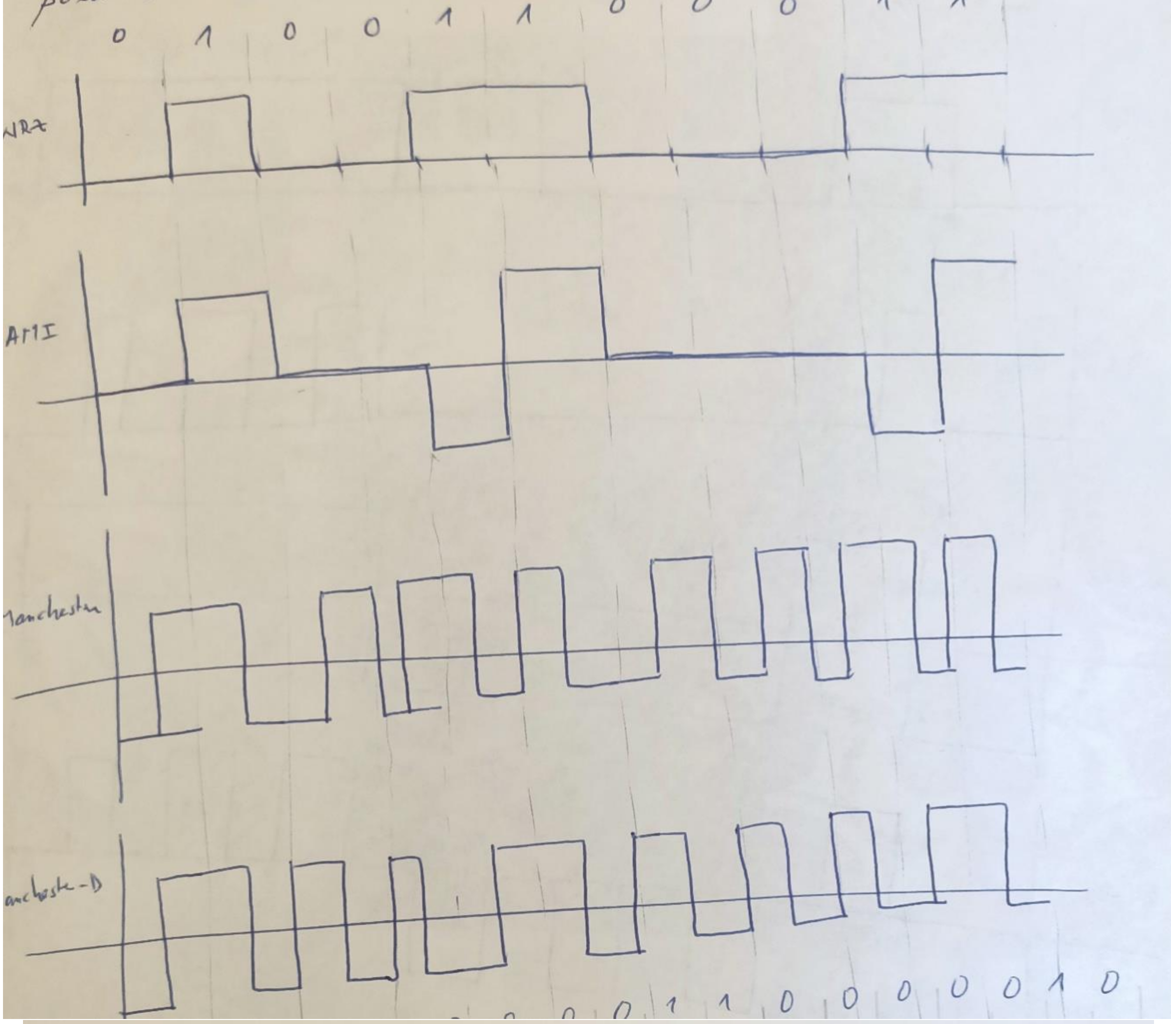
Solution feuille de TD N°01 en Communications numériques

Transmissions en bande de base

Exercice 01

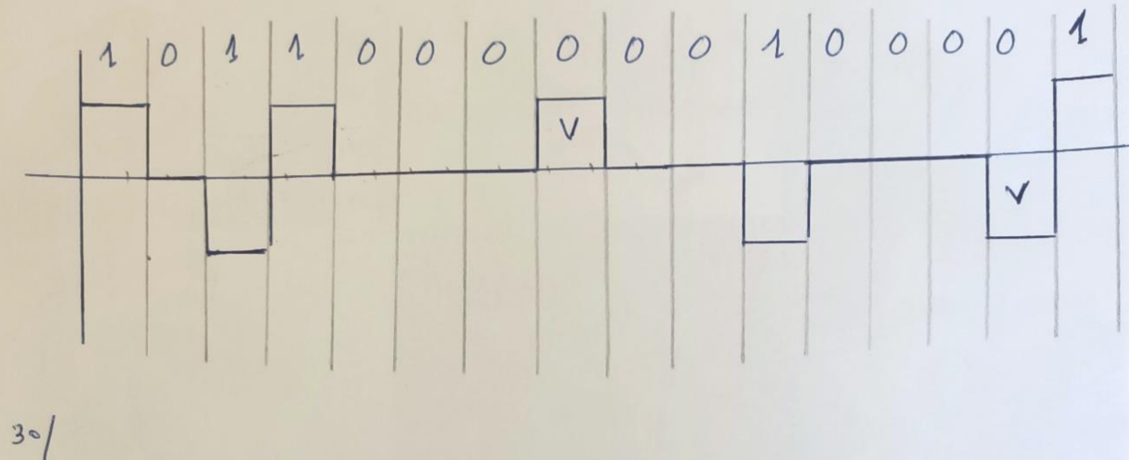


Solution Exo21

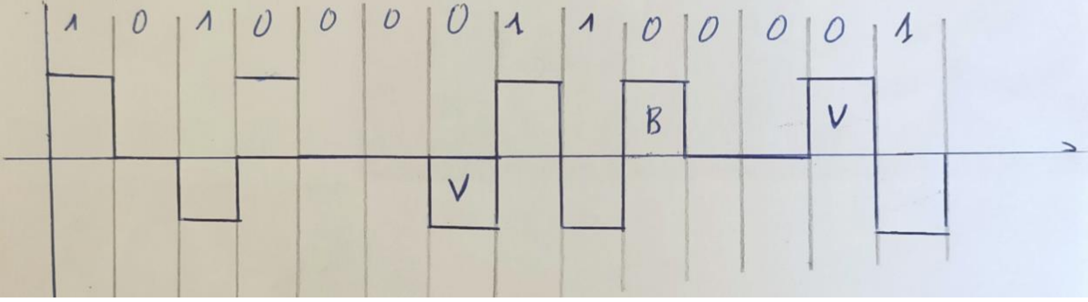


Exo 21

2°/ Code HDB3



3°/



Exercice 03

1- Manchester : 100101100
2- Manchester différentiel : 110111010

Solution de l'exo 4 :

1°/ Spectre de la fonction porte :

$$G(f) = TF\{g(t)\}$$

avec $g(t) = \begin{cases} A & si \ -\frac{T}{2} < t < \frac{T}{2} \\ 0 & ailleurs \end{cases}$

$$G(f) = \int_{-\frac{T}{2}}^{\frac{T}{2}} g(t)\, e^{-j2\pi f t}\, dt = \int_{-\frac{T}{2}}^{\frac{T}{2}} A\, e^{-j2\pi f t}\, dt$$

$$= \frac{A}{\pi f}\left[\frac{e^{j2\pi f \frac{T}{2}} - e^{-j2\pi f \frac{T}{2}}}{2j}\right] = \frac{AT}{\pi f T} \sin(\pi f T)$$

$$G(f) = AT\, sinc(\pi f T)$$

2°/ DSP du code NRZ bipolaire

Code NRZ bipolaire :

$1 \longrightarrow A$

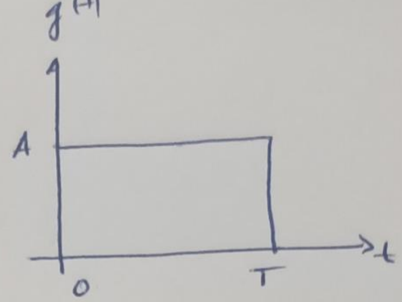
$0 \longrightarrow -A$

Formule de Benett :

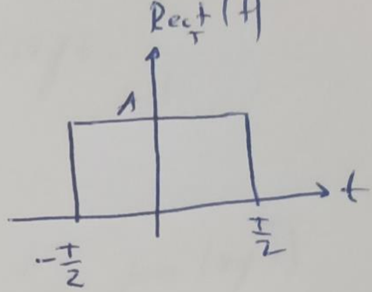
$$\gamma_{NRZ}(f) = \frac{1}{T} |G(f)|^2\, \gamma_{aa}(f)$$

avec $\gamma_{aa}(f) = \sigma_a^2 + \frac{\mu_a^2}{T} \sum_{i=-\infty}^{+\infty} \delta(f - \frac{i}{T})$

avec $G(f) = TF\{g(t)\}$



et $g(t) = \begin{cases} A & \text{si } 0 < t < T \\ 0 & \text{ailleurs} \end{cases}$



$g(t) = \mathrm{Rect}_T\left(t - \frac{T}{2}\right)$

avec $\mathrm{Rect}_T(t) = \begin{cases} A & \text{si } -\frac{T}{2} < t < \frac{T}{2} \\ 0 & \text{ailleurs} \end{cases}$

$TF\{\mathrm{Rect}_T(t)\} = AT \dfrac{\sin(\pi f T)}{\pi f T}$

donc $G(f) = AT \dfrac{\sin(\pi f T)}{\pi f T} e^{-j2\pi f \frac{T}{2}}$

et $|G(f)| = AT \dfrac{\sin(\pi f T)}{\pi f T}$

calcul de $\gamma_{aa}(f)$

$\mu_a = E[a_k] = \sum_k a_k P_{a_k}$ avec $a_k \in \{-1, 1\}$

avec $p(-1) = \frac{1}{2}$ et $p(1) = \frac{1}{2}$

donc $\mu_a = (1) \times \frac{1}{2} + (-1) \times \frac{1}{2} = 0$

$\sigma_a^2 = E[a_k^2] - \mu_a = \sum_k a_k^2 P_{a_k} - \mu_a$

$= \frac{1}{2}(1)^2 + \frac{1}{2}(-1)^2 = 1$

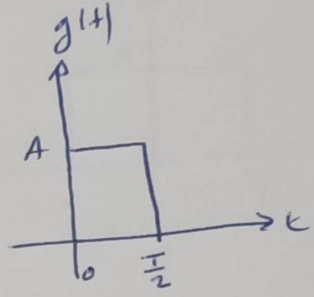
donc $\gamma_{aa}(f) = 1$

donc $\gamma_{NRZ}(f) = \frac{1}{T} |G(f)|^2 \times 1 = A^2 T \frac{\sin^2(\pi f T)}{(\pi f T)^2}$

b/ DSP du code RZ unipolaire.

Formule de Benett

$\gamma_{RZ}(f) = \frac{1}{T} |G(f)|^2 \, \gamma_{aa}(f)$



avec $g(t) = \begin{cases} A & \text{si } 0 < t < \frac{T}{2} \\ 0 & \text{ailleurs} \end{cases}$

donc $|G(f)| = A \frac{T}{2} \frac{\sin(\pi f \frac{T}{2})}{\pi f \frac{T}{2}}$

$\gamma_{aa}(f) = \sigma_a^2 + \frac{\mu_a^2}{T} \sum_i \delta(f - \frac{i}{T})$

$\mu_a = \sum_k a_k \, p(a_k)$ avec $a_k \in \{0, 1\}$

$\mu_a = 1 \times \frac{1}{2} + 0 \times \frac{1}{2} = \frac{1}{2}$

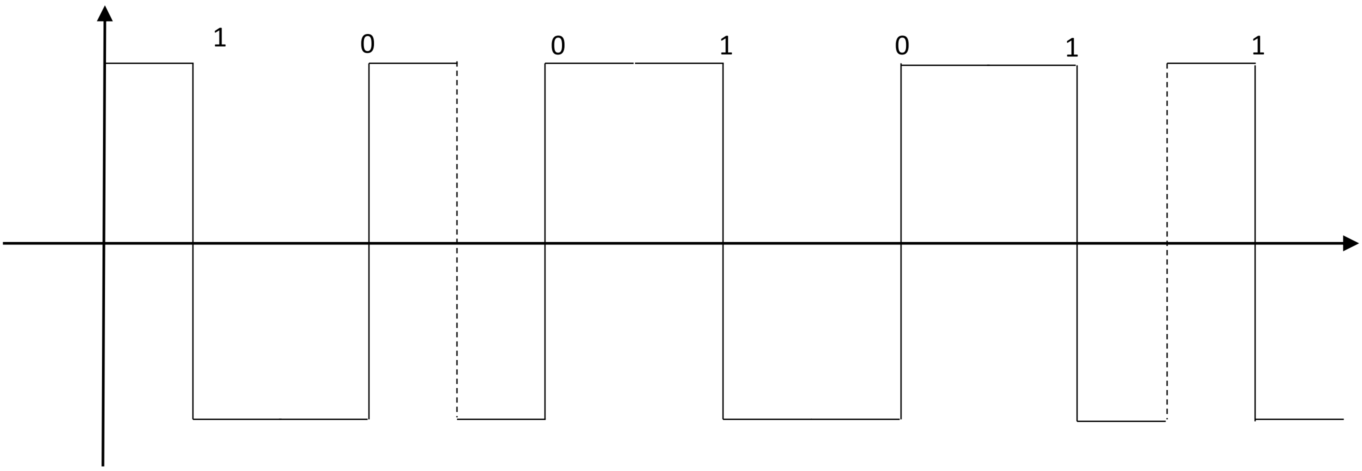
$\sigma_a^2 = E[a_k^2] - \mu_a^2 = \frac{1}{2} (1)^2 + \frac{1}{2} (0)^2 = \frac{1}{4}$

donc $\gamma_{aa}(f) = \frac{1}{4} + \frac{1}{4T} \sum \delta(f - \frac{i}{T})$

donc $\gamma_{RZ}(f) = \left[\frac{1}{4T} + \frac{1}{4T^2} \sum \delta(f - \frac{i}{T}) \right] |G(f)|^2$

Exercice 05

1. Code Manchester de la suite binaire I=1001011



2. DSP du code Manchester

$$x(t) = \sum_k a_k g(t - kT)$$

Formule de Benett

$$\boldsymbol{\gamma_{xx}(f) = \frac{1}{T}|G(f)|^2\gamma_{aa}(f)}$$

$$\boldsymbol{\gamma_{aa}(f) = \sigma_a^2 + \frac{\mu_a^2}{T}\sum_{i=-\infty}^{+\infty}\delta(f - \frac{i}{T})}$$

Avec $\boldsymbol{\mu_a = E[a_k] = \sum_k a_k p_{ak}}$ $avec\ a_k \epsilon\{-1,1\}$

Avec $p(1) = p(-1) = \frac{1}{2}$

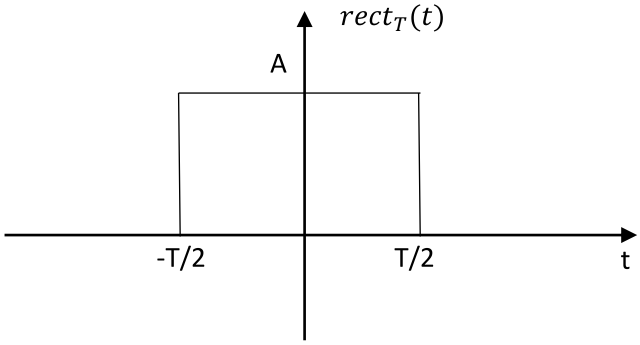
Donc $\mu_a = 1 \times \frac{1}{2} + (-1) \times \frac{1}{2} = \boldsymbol{0 = \mu_a}$

$$\boldsymbol{\sigma_a^2 = E[a_k^2] - \mu_{ak} = \sum_k a_k^2 p_{ak} - \mu_a^2}$$

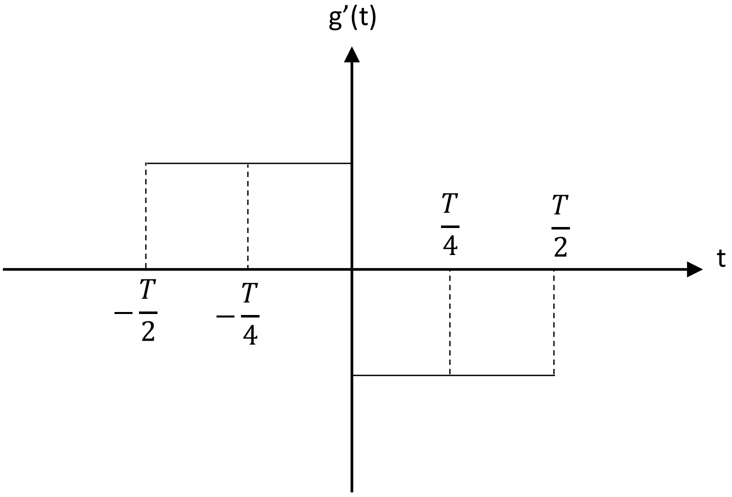
$$= \frac{1}{2} \times 1^2 + \frac{1}{2}(-1)^2 = \boldsymbol{1 = \sigma_a^2}$$

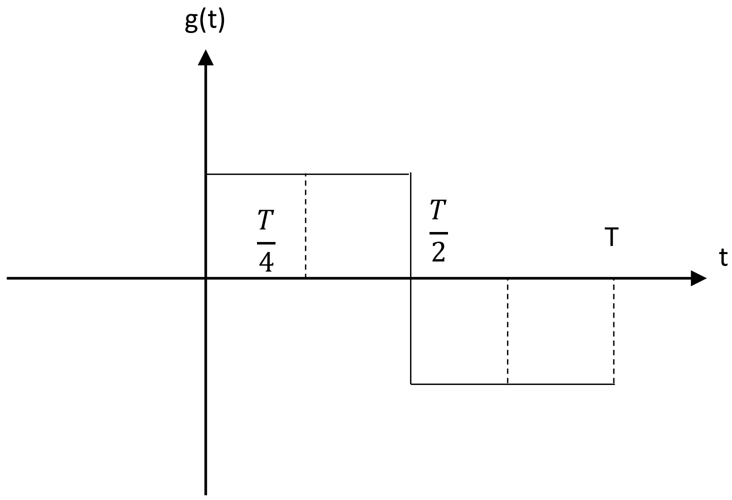
Donc $\boldsymbol{\gamma_{aa}(f) = 1}$

Calcul de G(f)



$$\boldsymbol{RECT(f) = AT\frac{\sin(\pi f T)}{\pi f T}}$$





$$g(t) = g'(t - \frac{T}{2})$$

$$G(f) = G'(f)\exp\left(j2\pi f\left(-\frac{T}{2}\right)\right)$$

$$g'(t) = rect_{\frac{T}{2}}\left(t + \frac{T}{4}\right) - rect_{\frac{T}{2}}\left(t - \frac{T}{4}\right)$$

Donc $G'(f) = \frac{AT}{2}\frac{\sin(\pi f\frac{T}{2})}{\pi f\frac{T}{2}}\left(\exp\left(j2\pi f\frac{T}{4}\right) - \exp\left(-j2\pi f\frac{T}{4}\right)\right)$

Donc $G(f) = \frac{AT}{2}\frac{\sin(\pi f\frac{T}{2})}{\pi f\frac{T}{2}}\left(\exp\left(j2\pi f\frac{T}{4}\right) - \exp\left(-j2\pi f\frac{T}{4}\right)\right)\exp(-j\pi fT)$

Or $\frac{\exp(j\alpha)-\exp(-j\alpha)}{2j} = \sin(\alpha)$

Donc $G(f) = \frac{AT}{2}\frac{\sin(\pi f\frac{T}{2})}{\pi f\frac{T}{2}}\frac{\left(\exp\left(j2\pi f\frac{T}{4}\right)-\exp\left(-j2\pi f\frac{T}{4}\right)\right)}{2j} \times 2j \times \exp(-j\pi fT)$

$$G(f) = \frac{AT}{2}\frac{\sin(\pi f\frac{T}{2})}{\pi f\frac{T}{2}}\sin(\pi f\frac{T}{2}) \times 2j \times \exp(-j\pi fT)$$

Donc $\boldsymbol{|G(f)| = AT\frac{\sin^2(\pi f\frac{T}{2})}{\pi f\frac{T}{2}}}$

Finalement,

$$\gamma_{xx}(f) = \frac{1}{T}|G(f)|^2\gamma_{aa}(f) = \frac{1}{T}A^2T^2\frac{\sin^4(\pi f\frac{T}{2})}{(\pi f\frac{T}{2})^2}\gamma_{aa}(f)$$

$$\boldsymbol{\gamma_{xx}(f) = A^2T\frac{\sin^4(\pi f\frac{T}{2})}{(\pi f\frac{T}{2})^2}}$$

Exercice 06

Symbole complexe peut s'écrire :

$$c_k = \mp 1 \mp j = \begin{cases} 1+j \\ 1-j \\ -1+j \\ -1-j \end{cases} \text{ avec } p(c_k) = \frac{1}{4}$$

$$\boldsymbol{\gamma_{xx}(f) = \frac{1}{T}|G(f)|^2\gamma_{cc}(f)}$$

$$\boldsymbol{\gamma_{cc}(f) = \sigma_c^2 + \frac{\mu_c^2}{T}\sum_{i=-\infty}^{+\infty}\delta(f-\frac{i}{T})}$$

Avec $\boldsymbol{\mu_c = E[c_k] = \sum_k c_k p_{ck}}$

$$\boldsymbol{\sigma_c^2 = E[c_k^2] - \mu_{ck} = \sum_k (c_k c_k^*) p_{ck} - \mu_c^2}$$

$$\mu_c = E[c_k] = \sum_k c_k p_{ck}$$

$$= (1+j)\frac{1}{4} + (1-j)\frac{1}{4} + (-1+j)\frac{1}{4}$$

$$+ (-1-j)\frac{1}{4} = \mathbf{0 = \mu_c}$$

$\sigma_c^2 = \sum_k (c_k c_k^*) p_{ak} - \mu_c^2 = (1+j)(1-j)\frac{1}{4} + (1-j)(1+j)\frac{1}{4} + (-1+j)(-1-j)\frac{1}{4} + (-1-j)(-1+j)\frac{1}{4} = \mathbf{2 = \sigma_c^2}$

Donc $\boldsymbol{\gamma_{cc}(f) = 2}$

$$H(f) = \boldsymbol{AT\frac{\sin(\pi f T)}{\pi f T}}$$

Donc $\gamma_{xx} = \frac{1}{T}[AT sinc(\pi f T)]^2 \cdot 2 = \boldsymbol{2A^2 T sinc^2(\pi f T) = \gamma_{xx}(f)}$