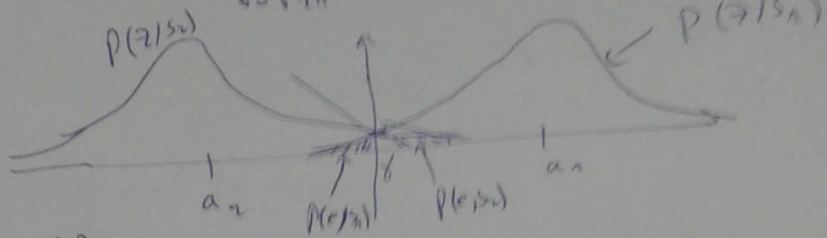
Corrigé TD2

TD 2 :

Exo 4 :

1] $p(z|s_1) = \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z-a_1}{\sigma_0}\right)^2\right]$

$p(z|s_2) = \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z-a_2}{\sigma_0}\right)^2\right]$



2] ① MAP :

$p(s_1|z) > p(s_2|z)$ on décide H_1

$p(s_2|z) > p(s_1|z)$ on décide H_2

$$p(s_1|z) \underset{H_2}{\overset{H_1}{\gtrless}} p(s_2|z)$$

② bayes : $p(s_i|z) = \frac{p(z|s_i)\,p(s_i)}{p(z)}$

$$\frac{p(z|s_1)\,p(s_1)}{p(z)} \underset{H_2}{\overset{H_1}{\gtrless}} \frac{p(z|s_2)\,p(s_2)}{p(z)}$$

$$\frac{p(z|s_1)}{p(z|s_2)} \underset{H_2}{\overset{H_1}{\gtrless}} \frac{p(s_2)}{p(s_1)} \qquad p(s_1) = p(s_2) = \frac{1}{2}$$

$$\Rightarrow \frac{p(z|s_1)}{p(z|s_2)} \underset{H_2}{\overset{H_1}{\gtrless}} 1$$

③ Remplaçant les PDFs :

$$\frac{\frac{1}{\sigma_0\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{z-a_1}{\sigma_0}\right)^2\right)}{\frac{1}{\sigma_0\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{z-a_2}{\sigma_0}\right)^2\right)} \underset{H_2}{\overset{H_1}{\gtrless}} 1$$

$$Exp\left[\frac{1}{2}\left(\frac{z-a_2}{\sigma_0}\right)^2 - \frac{1}{2}\left(\frac{z-a_1}{\sigma_0}\right)^2\right] \underset{H_2}{\overset{H_1}{\gtrless}} 1$$

$$Exp\left[\frac{1}{2\sigma_0^2}\left(z^2 + a_2^2 - 2za_2 - z^2 - a_1^2 + 2za_1\right)\right] \underset{H_2}{\overset{H_1}{\gtrless}} 1$$

$$Exp\left[\frac{1}{2\sigma_0^2}\left(2(a_1-a_2)z - (a_1^2 - a_2^2)\right)\right] \underset{H_2}{\overset{H_1}{\gtrless}} 1$$

$$Exp\left[\frac{z(a_1-a_2)}{\sigma_0^2} - \frac{a_1^2 - a_2^2}{2\sigma_0^2}\right] \underset{H_2}{\overset{H_1}{\gtrless}} 1$$

(4) Appliquant le log :

$$\frac{z(a_1-a_2)}{\sigma_0^2} - \frac{a_1^2 - a_2^2}{2\sigma_0^2} \underset{H_2}{\overset{H_1}{\gtrless}} 0$$

$$\frac{z(a_1-a_2)}{\sigma_0^2} \underset{H_2}{\overset{H_1}{\gtrless}} \frac{(a_1+a_2)(a_1-a_2)}{2\sigma_0^2}$$

$$\boxed{z \underset{H_2}{\overset{H_1}{\gtrless}} \frac{a_1+a_2}{2} = \gamma}$$ la règle de décision ML

3) P_B : $P_B = P(e, S_1) + P(e, S_2) = P(e|S_1)\cdot P(S_1) + P(e|S_2)P(S_2)$

$P(S_1) = P(S_2) = \frac{1}{2} \Rightarrow P_B = \frac{1}{2}P(e|S_1) + \frac{1}{2}P(e|S_2) = P(e|S_1) = P(e|S_2)$ Symétrie

$P(e|S_1) = P(H_2|S_1)$

$P(e|S_2) = P(H_1|S_2)$

$$P_B = \int_{-\infty}^{\gamma} p(z|H_1)\,dz = \int_{\gamma}^{+\infty} \frac{1}{\sigma_0\sqrt{2\pi}} exp\left(-\frac{1}{2}\left(\frac{z-a_2}{\sigma_0}\right)^2\right] dz$$

$$Q(x) = \int_{x}^{+\infty} \frac{1}{\sqrt{2\pi}} exp\left(-\frac{u^2}{2}\right] du$$

on pose : $\frac{z-a_2}{\sigma_0} = u \Rightarrow dz = \sigma_0\, du$

$z = \gamma = \frac{a_1+a_2}{2}$

$u = \frac{z-a_2}{\sigma_0} = \frac{\frac{a_1+a_2}{2} - a_2}{\sigma_0} = \frac{a_1-a_2}{2\sigma_0}$

Suite exo 4 :

$$P_B = \int_{\frac{a_1+a_2}{2\sigma_0}}^{+\infty} \frac{1}{\sigma_0\sqrt{2\pi}} \exp\left(-\frac{1}{2}u^2\right) \sigma_0 \, du$$

$$\boxed{P_B = Q\left(\frac{a_1 - a_2}{2\sigma_0}\right)}$$

$$d_{12} = a_1 - a_2$$

$$\sigma_0^2 = \frac{N_0}{2} \Rightarrow \sigma_0 = \sqrt{\frac{N_0}{2}}$$

$$P_B = Q\left(\frac{d_{12}}{2\sqrt{\frac{N_0}{2}}}\right) = Q\left(\sqrt{\frac{d_{12}^2}{4\frac{N_0}{2}}}\right) = \boxed{Q\left(\sqrt{\frac{d_{12}^2}{2N_0}}\right)}$$

A.N : $P_B = Q\left(\sqrt{\frac{4^2}{2 \times 2}}\right) = Q(2) =$

$$\boxed{P_B = 0{,}02275}$$

Exo3 :

1] $n = \log_2 M = \log_2 8 = 3$ bits / Symbole.

2] $B \geq \frac{1+\alpha}{2T_s} = \frac{1+\alpha}{2(3T_b)} = \frac{1+\alpha}{6T_b} = \left(\frac{1+\alpha}{6}\right) D_b$

$D_b \leq \frac{6B}{1+\alpha} \Rightarrow D_{b\,max} = \frac{6B}{1+\alpha} = \frac{6 \times 70 \times 10^3}{1+0,4} = 300$ Kbits/s

3] $P_{es} = 10^{-5}$

$D_b = 300$ Kbits/s

$D_b = 3 \times 10^5$ bits/s $\longrightarrow \frac{E_b}{N_0} = 15$ dB $\rightarrow P_{es} = 2 \cdot 10^{-3}$

Donc à 300 Kbits/s on ne peut pas atteindre les performances demandées ($P_{es} = 10^{-5}$).

$D_b' \longrightarrow$ pour $P_{es} = 10^{-5} \Rightarrow \frac{E_b}{N_0} = 18$ dB

$\Rightarrow D_b' = 1,5 \cdot 10^5$ bits/s $= 150$ Kbits/s

$D_b' = 150$ Kbits/s $< D_{b\,max} = 300$ Kbits/s

Donc on peut transmettre sans IES à 150 bits/s