AKHRIB
FATEH

TD MMC.

(20/11/2011)

Application 1

Soit l'état de σ définit par le tenseur de contraintes suivant:

$$[\sigma] = \begin{bmatrix} -5a & -4a & 0 \\ -4a & a & 0 \\ 0 & 0 & a \end{bmatrix} \qquad \tau_{xz} = \tau_{yz} = 0 \quad \sigma_z = a = \sigma_{minimale}$$

1) Déterminer analytiquement les contraintes principale et les plans principaux corespondant:

a) <u>Equation cubique est:</u>

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0.$$

$$I_1 = \sigma_x + \sigma_y + \sigma_z = -5a + a + a = -3a$$

$$I_2 = \sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{xz}^2 - \tau_{zx}^2 = -25a^2$$

$$I_3 = \begin{vmatrix} -5a & -4a & 0 \\ -4a & a & 0 \\ 0 & 0 & a \end{vmatrix} = -5a\begin{vmatrix} a & 0 \\ 0 & a \end{vmatrix} + 4a\begin{vmatrix} -4a & 0 \\ 0 & a \end{vmatrix} + 0\begin{vmatrix} -4a & a \\ 0 & 0 \end{vmatrix}$$

$$= -21a^3$$

$$\Rightarrow \sigma^3 + 3a\sigma^2 - 25a^2\sigma + 21a^3 = 0. \longrightarrow ①$$

On déja une facette principale correspondant à σ_z car $\tau_{yz} = \tau_{xz} = 0$, donc $\boxed{\sigma_z = a}$ est une contrainte principale.

C'est-à-dire que nous avons déja une racine

$\sigma' = a$, Alors l'équation cubique peut être écrire sous la forme.

$(\sigma - a)(A\sigma^2 + B\sigma + C) = 0$.

$A\sigma^3 + B\sigma^2 + C\sigma - Aa\sigma^2 - Ba\sigma - Ca = 0$.

$A\sigma^3 + (B - Aa)\sigma^2 + (C - Ba)\sigma - Ca = 0$

$\sigma^3 + 3a\sigma^2 - 25a^2\sigma + 21a^3 = 0$

$$\begin{cases} A = 1 \\ B - Aa = 3a \\ C - Ba = -25a^2 \\ -Ca = 21a^3 \end{cases} \Rightarrow \begin{cases} A = 1 \\ B = 4a \\ \\ C = -21a^2 \end{cases}$$

$\Rightarrow (\sigma - a)(\sigma^2 + 4a\sigma - 21a^2) = 0$. ——— ②

$\sigma^2 + 4a\sigma - 21a^2 = 0 \Rightarrow \sqrt{\Delta} = 10a$.

$\sigma'' = \dfrac{-4a - 10a}{2} = -7a$

$\sigma''' = \dfrac{-4a + 10a}{2} = 3a$.

on a : $\sigma_1 > \sigma_2 > \sigma_3$

$$\begin{cases} \sigma_1 = 3a \\ \sigma_2 = a \\ \sigma_3 = -7a \end{cases}$$

b) Les cosinus directeur :

$$\begin{cases} (\sigma_x - \sigma_i) l_i + \tau_{xy} m_i + \tau_{xz} n_i = 0 \\ \tau_{xy} l_i + (\sigma_y - \sigma_i) m_i + \tau_{yz} n_i = 0 \\ \tau_{xz} l_i + \tau_{yz} m_i + (\sigma_z - \sigma_i) n_i = 0 \end{cases}$$

$i = 1 \Rightarrow \sigma_1 = 3a$

$$\begin{cases} (-5a-3a)\,l_1 - 4a\,m_1 + 0\,n_1 = 0 \\ -4a\,l_1 + (a-3a)\,m_1 + 0\cdot n_1 = 0 \\ 0\,l_1 + 0\,m_1 + (a-3a)\,n_1 = 0 \end{cases} \Rightarrow \begin{cases} 8al_1 + 4am_1 = 0 \\ 4al_1 + 2am_1 = 0 \\ 2am_1 = 0 \end{cases} (3)$$

$l_1^2 + m_1^2 + n_1^2 = 1 \quad \ldots (4)$

$$\begin{cases} 2al_1 + am_1 = 0 \\ 2n_1 = 0 \\ l_1^2 + m_1^2 + n_1^2 = 1 \end{cases} \Rightarrow \begin{cases} m_1 = -2l_1 \\ n_1 = 0 \\ 5l_1^2 = 1 \end{cases} \Rightarrow \boxed{\begin{cases} l_1 = \pm \frac{1}{\sqrt5} \\ m_1 = \pm \frac{2}{\sqrt5} \\ n_1 = 0 \end{cases}}$$

On proced de la m̃ manière pour les direction 2 et 3 ce qui donne :

$$\vec{v}_2 \begin{cases} l_2 = 0 \\ m_2 = 0 \\ n_2 = \pm 1 \end{cases} \quad \text{et} \quad \vec{v}_3 \begin{cases} l_3 = \pm \frac{2}{\sqrt5} \\ m_3 = \pm \frac{1}{\sqrt5} \\ n_3 = 0 \end{cases}$$

Application n°2 :

-) Angles :

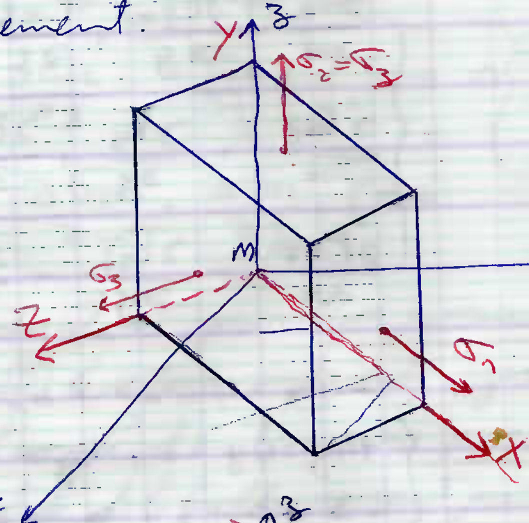
$$\begin{cases} l_1 = \cos\alpha \Rightarrow \alpha_1 = 63{,}43 \pm 180° \\ m_1 = \cos\beta \Rightarrow \beta_1 = 26{,}57 \pm 180° \\ n_1 = \cos\gamma \Rightarrow \gamma_1 = 90° \pm 180° \end{cases}$$

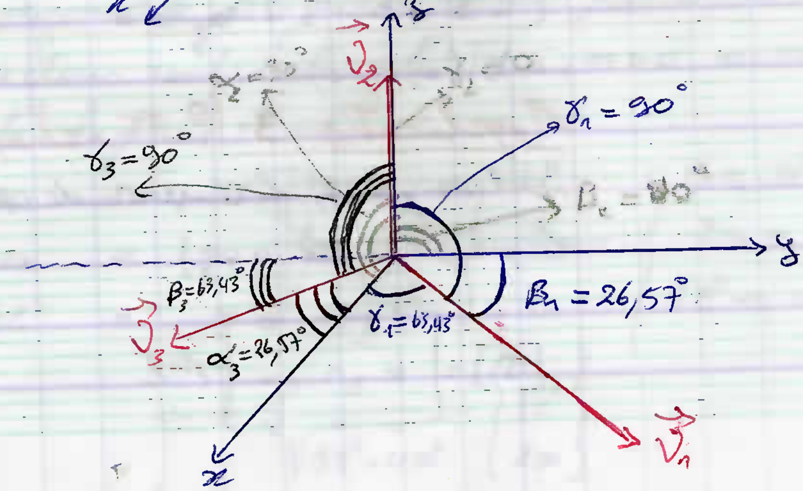
$$\begin{cases} l_2 = \cos\alpha \Rightarrow \alpha_2 = 90 \pm 180 \\ m_2 = \cos\beta \Rightarrow \beta_2 = 90 \pm 180 \\ n_2 = \cos\gamma \Rightarrow \gamma_2 = 0 \pm 180 \end{cases}$$

$$\begin{cases} l_3 = \cos\alpha \Rightarrow \alpha_3 = 26,57^\circ \pm 180^\circ \\ m_3 = \cos\beta \Rightarrow \beta_3 = 63,43^\circ \pm 180^\circ \\ n_3 = \cos\gamma \Rightarrow \gamma_3 = 90^\circ \pm 180^\circ \end{cases}$$

on vérifie que $26,57^\circ + 63,43^\circ = 90^\circ$.

→ On constate que les plans principaux sont bien orthogonaux et que la contrainte σ_z est bien principale et qu'elle correspond à σ_2, comme on l'a vue analytiquement.





Application 2 :

Soit le tenseur de contrainte σ_{ij} :

$\sigma_x = -2x^2 + 3y^2 - 5z$ $\qquad \tau_{xy} = z + 4xy - 7$

$\sigma_y = -2y^2$ $\qquad \tau_{xz} = -3x + y + 1$

$\sigma_z = 3x + y + 3z - 5$ $\qquad \tau_{yz} = 0$

$b(b_x, b_y, b_z) = ?$ → état d'équilibre statique

→

$$\begin{cases} \dfrac{\partial \sigma_x}{\partial x} + \dfrac{\partial \tau_{yx}}{\partial y} + \dfrac{\tau_{xz}}{\partial z} + b_x = 0 \\[2ex] \dfrac{\partial \tau_{xy}}{\partial x} + \dfrac{\partial \sigma_y}{\partial y} + \dfrac{\partial \tau_{yz}}{\partial z} + b_y = 0 \\[2ex] \dfrac{\partial \tau_{xz}}{\partial x} + \dfrac{\partial \tau_{yz}}{\partial y} + \dfrac{\partial \sigma_z}{\partial z} + b_z = 0 \end{cases}$$

$$\begin{cases} -4x + 4x + 0 + b_x = 0 \\ +4y - 4y + 0 + b_y = 0 \\ -3 + 0 + 3 + b_z = 0 \end{cases} \Rightarrow \boxed{\begin{cases} b_x = 0 \\ b_y = 0 \\ b_z = 0 \end{cases}}$$

$b(0, 0, 0)$

Application 3 : (P 25 - chapitre 2).

Soit au pt (P) le tenseur de contrainte $[\sigma]$ dans le repère (x, y, z).

$$[\sigma] = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \text{MPa}$$

1) Contrainte principale :

Application :

$$[\sigma] = \begin{bmatrix} 0 & -2x & 3z^2 \\ -2x & 0 & 0 \\ 3z^2 & 0 & 0 \end{bmatrix} N/m^2$$

1) Les composante des forces volumique :

$$\begin{cases} \dfrac{\partial \sigma_{11}}{\partial x} + \dfrac{\partial \sigma_{12}}{\partial y} + \dfrac{\partial \sigma_{13}}{\partial z} + f_1 = 0. \\ \dfrac{\partial \sigma_{21}}{\partial x} + \dfrac{\partial \sigma_{22}}{\partial y} + \dfrac{\partial \sigma_{23}}{\sigma z} + f_2 = 0 \\ \dfrac{\partial \sigma_{31}}{\partial x} + \dfrac{\partial \sigma_{32}}{\partial x} + \dfrac{\partial \sigma_{33}}{\partial z} + f_3 = 0. \end{cases} \Rightarrow \begin{cases} f_1 = -6z \\ f_2 = 2 \quad N/m^3 \\ f_3 = 0 \end{cases}$$

2) Détermination, par leur normales, les facettes sur lesquelle le vecteur contrainte agissant est nule, et l'angle d'inclinaison par rapport à l'horizontale de la facette passant par le point $M(1,0,1)$:

$\rightarrow$ vecteur contrainte nule : $T = \sigma \cdot n = 0$, n : normale à la facette.

$$\begin{bmatrix} 0 & -2x & 3z^2 \\ -2x & 0 & 0 \\ 3z^2 & 0 & 0 \end{bmatrix} \begin{Bmatrix} n_1 \\ n_2 \\ n_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \text{ donne : } \begin{cases} -2x n_2 + 3z^2 n_3 = 0 \\ -2x n_1 = 0 \\ 3z^2 n_1 = 0 \end{cases}$$

$$\Rightarrow n = \frac{\pm 1}{\sqrt{9z^4 + 4x^2}} \begin{Bmatrix} 0 \\ 3z^2 \\ 2x \end{Bmatrix}.$$

→ Angle d'inclinaison au point M (1,0,1) :

La normale est : $n = \frac{1}{\sqrt{3}} \begin{Bmatrix} 0 \\ 3 \\ 2 \end{Bmatrix}$.

L'angle entre la normale et oy est :

$$\alpha = \arccos(n, [0\ 1\ 0]) = 33,7°$$

Donc l'angle entre la facette et le plan horizontale est : $\beta = 90 - \alpha = 56,3°$.

3) <u>Les contrainte les contrainte principale</u>

$\sigma_1 = -\sqrt{4x^2 + 9z^2}$, $\sigma_2 = 0$, $\sigma_3 = \sqrt{4x^2 + 9z^2}$.

4) <u>Etat pur de contrainte correspond a cas :</u>

On a 2 contraintes égales de signes opposés et une contrainte nulle, ce qui correspond au cas de cisaillement pure.

Application :

31/10/2011

Exo1 :

Les expression suivantes sont elles juste ou fausses (justifie)

1) $a_{ij} x_i x_j + b_i = 0$ → fausse (i)

2) $a_{ij} c_{jj} = e_i$ → fausse (j)

3) $w = \int T_{ij} \, de_{ij}$ → juste

4) $b_{ij} = \frac{\partial x_i}{\partial x_k} L_{ij} + \frac{\partial x_j}{\partial x_k} L_{ki}$ → fausse (k, i)

5) $da_i = J \frac{\partial x_m}{\partial x_i} dA_m$ → juste

6) $dS_i = \sqrt{dx_i dx_j}$ → fausse (j)

(ii) indice muet / Sommation 2 fois dans le monome $\Rightarrow \sum_{i=1}^{3}$ monome

(i) indice libre → apparait (1) fois dans la monome ↳ c'est le n° de l'equation.

$\underbrace{a_{ii} b_j}_{\text{monome}} + c = 0$

Exo2 :

$U_i = (3, 2, 2)$, $v_i = (1, 4, 1)$ $\quad a_{ij} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$

1) $a_i = a_{im} U_m$ $\quad$ 2) $b_{ij} = U_i v_j$

3) $C_{ij} = a_{ij} + b_{ij}$ $\quad$ 4) $\alpha = a_{mm}$

28/11/2011 **Application :**

Soit au pt (P) le tenseur de contrainte $[\sigma]$ dans le repe
(x, y, z)

$$[\sigma] = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

1) Determiner les contraintes principale puis les direction principale corespondant ?

2) Ecrire le tenseur de contrainte dans le repere princip ale, puis verifier que les invariant (I_i) sont les même que ~~seux~~ du tenseur initial ?

1) → Contrainte principale :

$$\sigma^3 - I_1 \sigma^2 + I_2 \sigma - I_3 = 0$$

$$\begin{cases} I_1 = \sigma_x + \sigma_y + \sigma_z = 1 + 1 + 3 = 5 \\ I_2 = \sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_z \sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{xz}^2 \\ I_2 = 1 + 3 + 3 - 1 - 0 - 0 \Rightarrow I_2 = 6 \end{cases}$$

$$I_3 = \begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{vmatrix} \Rightarrow \sigma_x \begin{vmatrix} \sigma_y & \tau_{yz} \\ \tau_{zy} & \sigma_z \end{vmatrix} - \tau_{xy} \begin{vmatrix} \tau_{yx} & \tau_{yz} \\ \tau_{zx} & \sigma_z \end{vmatrix} + \tau_{xz} \begin{vmatrix} \tau_{yx} & \\ \tau_{zx} & \end{vmatrix}$$

$$I_3 = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{vmatrix} = 0$$

$$\Rightarrow \boxed{\sigma^3 - 5\sigma^2 + 6\sigma = 0}$$

$$(\sigma - 0)(A\sigma^2 + b\sigma + c) = 0$$

$$\sigma^3 - 5\sigma^2 + 6\sigma = 0 \Rightarrow \sigma(\sigma^2 - 5\sigma + 6) = 0$$

$$(\sigma - 0)(\sigma^2 - 5\sigma + 6) = 0 \Rightarrow \sigma' = 0$$

on superpose : $\begin{cases} A = 1 \\ B = -5 \\ C = 6 \end{cases}$

$$\sigma^2 - 5\sigma + 6 = 0 \Rightarrow \Delta = 25 - 24 = 1 \Rightarrow \sqrt{\Delta} = 1$$

$$\sigma'' = \frac{+5-1}{2} = \frac{4}{2} = 2, \quad \sigma''' = \frac{+5+1}{2} = 3$$

on a : $\sigma_1 > \sigma_2 > \sigma_3$

$$\Rightarrow \boxed{\begin{cases} \sigma_1 = 3 \\ \sigma_2 = 2 \\ \sigma_3 = 0 \end{cases}}$$

→ Direction principale :

$$\begin{cases} (\sigma_x - \sigma_i)\, l_i + \tau_{xy}\, m_i + \tau_{xz}\, n_i = 0 \\ \tau_{yx}\, l_i + (\sigma_y - \sigma_i)\, m_i + \tau_{yz}\, n_i = 0 \\ \tau_{zx}\, l_i + \tau_{zy}\, m_i + (\sigma_z - \sigma_i)\, n_i = 0 \end{cases}$$

$$\begin{bmatrix} (\sigma_x - \sigma_i) & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & (\sigma_y - \sigma_i) & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & (\sigma_z - \sigma_i) \end{bmatrix} \begin{bmatrix} l_i \\ m_i \\ n_i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$i = 1 : \sigma_1 = 3.$

$$\begin{bmatrix} (1-3) & 1 & 0 \\ 1 & (1-3) & 0 \\ 0 & 0 & (3-3) \end{bmatrix} \begin{bmatrix} l_1 \\ m_1 \\ n_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -2l_1 + m_1 + 0n_1 = 0 \\ l_1 - 2m_1 + 0n_1 = 0 \\ 0l_1 + 0m_1 + 0n_1 = 0 \end{cases} \Rightarrow \begin{cases} -2l_1 + m_1 = 0 \\ l_1 - 2m_1 = 0 \\ 0n_1 = 0 \end{cases}$$

$$\Rightarrow l_1^2 + m_1^2 + n_1^2 = 1$$

$$\sigma_1 = 3 \rightarrow \Rightarrow \begin{cases} l_1 = 0 = \cos\alpha \Rightarrow \alpha = 90 \pm 180 \\ m_1 = 0 = \cos\beta \Rightarrow \beta = 90 \pm 180 \\ n_1 = \pm 1 = \cos\gamma \Rightarrow \gamma = 0 \pm 180 \end{cases}$$

$i = 2,\ \sigma_2 = 2.$

$$\sigma_2 = 2 \rightarrow \begin{cases} l_2 = \pm 1/\sqrt{2} = \cos\alpha \Rightarrow \alpha = 45 \pm 180 \\ m_2 = \pm 1/\sqrt{2} = \cos\beta \Rightarrow \beta = 45 \pm 180 \\ n_2 = 0 = \cos\gamma \Rightarrow \gamma = 90 \pm 180 \end{cases}$$

$i = 3,\ \sigma_3 = 0$

$$\sigma_3 = 0 \rightarrow \begin{cases} l_3 = \pm 1/\sqrt{2} = \cos\alpha \Rightarrow \alpha = 45 \pm 180 \\ m_3 = \pm 1/\sqrt{2} = \cos\beta \Rightarrow \beta = 45 \pm 180 \\ n_3 = 0 = \cos\gamma \Rightarrow \gamma = 90 \pm 180 \end{cases}$$

2) Calcul le tenseur :

$$[\sigma]_p = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} I_1 = 3+2 = 5 \\ I_2 = 6 \\ I_3 = 0 \end{cases}$$

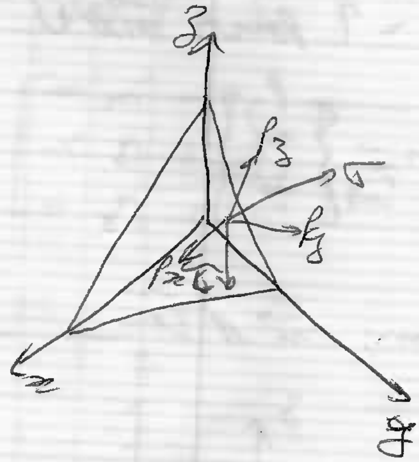
quelque soit des contraintes on avoir ses Invariants.

Ex n°2 ✓

$P_x = 1400 \, Kg/cm^2$

$P_y = -400 \, Kg/cm^2$

$P_z = -707 \, Kg/cm^2$



le plan considéré est définie par les ~~contraintes (cosinus~~ ~~s)~~ directeurs suivant
$$\begin{cases} l = \frac{+1}{2} \\ m = \frac{1}{2} \\ n = \frac{1}{\sqrt{2}} \end{cases}$$

claculer les contraintes σ et τ sur le plan considere

$$\begin{cases} P_x = l\sigma_x + m\tau_{yx} + n\tau_{zx} \\ P_y = l\tau_{xy} + m\sigma_y + n\tau_{zy} \\ P_z = l\tau_{xz} + m\tau_{yz} + n\sigma_z \end{cases}$$

$\vec{P} = \vec{P_x} + \vec{P_y} + \vec{P_z} = \vec{\sigma} + \vec{\tau}$

$P_x^2 + P_y^2 + P_z^2 = \sigma^2 + \tau^2$

$\sigma = P_x l + P_y m + P_z n$.

$\tau = \sqrt{P_x^2 + P_y^2 + P_z^2 - \sigma^2}$

$\boxed{\sigma = 0{,}075 \, Kg/cm^2}$

$\boxed{\tau = 1618{,}59 \, Kg/cm^2}$

21/11/2012

Exo n°1 : V

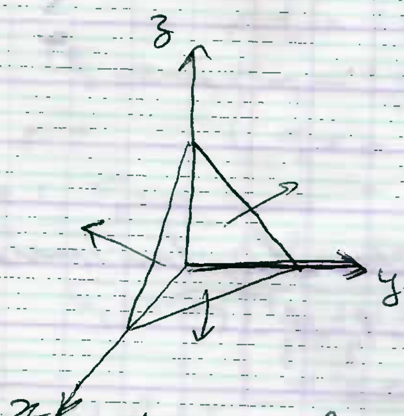
Structure

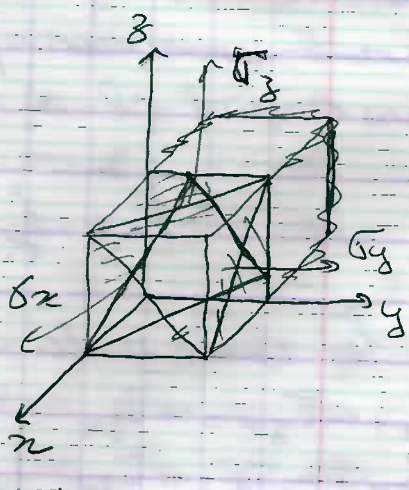
$$[\sigma] = \begin{bmatrix} 10 & -5 & 0 \\ -5 & 8 & 3 \\ 0 & 3 & 0 \end{bmatrix} \text{ MPa}$$

$\vec{t} = ?$ pour $\vec{n} \parallel \vec{u}$ telque $\vec{u} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$

$$n = \begin{Bmatrix} l \\ m \\ n \end{Bmatrix} \begin{matrix} x \\ y \\ z \end{matrix}$$

on ; $\vec{n} \parallel \vec{u}$





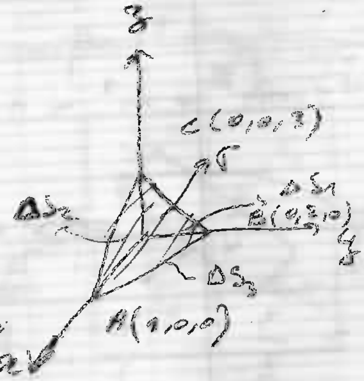
$$t_x = \sigma_x \, l + \tau_{yx} \, m + \tau_{zx} \, n$$
$$t_y = \tau_{xy} \, l + \sigma_y \, m + \tau_{zy} \, n$$
$$t_z = \tau_{xz} \, l + \tau_{yz} \, m + \sigma_z \, n$$

$$t = \begin{Bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{Bmatrix} \begin{Bmatrix} l \\ m \\ n \end{Bmatrix}$$

$$\vec{n} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{\sqrt{2^2+3^2+1^2}} \begin{Bmatrix} 2 \\ 3 \\ 1 \end{Bmatrix} = \frac{1}{\sqrt{14}} \begin{Bmatrix} 2 \\ 3 \\ 1 \end{Bmatrix}$$

$$\{t\} = [\sigma]\{n\} \Rightarrow \begin{Bmatrix} t_x \\ t_y \\ t_z \end{Bmatrix} = \begin{bmatrix} 10 & -5 & 0 \\ -5 & 3 & 3 \\ 0 & 3 & 0 \end{bmatrix} \begin{Bmatrix} 2 \\ 3 \\ 1 \end{Bmatrix} \frac{1}{\sqrt{14}}$$

$$= \frac{1}{\sqrt{14}} \begin{Bmatrix} 5 \\ 17 \\ 9 \end{Bmatrix} \text{ MPa}.$$



<u>Exo 2 :</u>

$$[\sigma] = \begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$\vec{t} = ?$ sur un plan // à la facette ABC

$\sigma = ?$, $\tau = ?$

$$\{t\} = [\sigma] \cdot \{n\} \Rightarrow \begin{Bmatrix} t_x \\ t_y \\ t_z \end{Bmatrix} = \begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \frac{1}{\sqrt{14}}$$

$$\{t\} = \begin{Bmatrix} -2 \\ 7 \\ 8 \end{Bmatrix} \frac{1}{\sqrt{14}}$$

$$\sigma^3 - I_1 \sigma^2 + I_2 \sigma - I_3 = 0$$

$$\begin{cases} I_1 = \sigma_x + \sigma_y + \sigma_z = 7 \\ I_2 = \underbrace{\sigma_x \sigma_y}_{6} + \underbrace{\sigma_x \sigma_z}_{4} + \underbrace{\sigma_y \sigma_z}_{6} - \underbrace{\tau_{xy}^2}_{4} - \underbrace{\tau_{xz}^2}_{0} - \underbrace{\tau_{yz}^2}_{1} = 11 \end{cases}$$

$$I_3 = \begin{vmatrix} 2 & -2 & 0 \\ -2 & 3 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 2(5) - 2(-4) = 18$$

$$\vec{n} = ? \qquad n \begin{Bmatrix} \ell \\ m \\ n \end{Bmatrix}$$

$$\ell = \frac{\Delta s_1}{\Delta s} \qquad m = \frac{\Delta s_2}{\Delta s} \qquad n = \frac{\Delta s_3}{\Delta s} \qquad \left| \begin{array}{l} \Delta s_1 = \frac{2 \times 3}{2} = 3 \\ \Delta s_2 = \frac{1 \times 3}{2} = \frac{3}{2} \\ \Delta s_3 = 1 \end{array} \right.$$

$$\sqrt{\ell^2 + m^2 + n^2} = 1 \Rightarrow \sqrt{\left(\frac{3}{\Delta s}\right)^2 + \left(\frac{3}{2}\right)^2 \frac{1}{\Delta s^2} + \left(\frac{1}{\Delta s}\right)^2} = 1$$

$$\sqrt{9 + \frac{9}{4} + 1} = \Delta s \Rightarrow \Delta s = \frac{7}{2}$$

$$\ell = \frac{3}{\frac{7}{2}} = \frac{6}{7}, \quad m = \frac{3}{7}, \quad n = \frac{2}{7} \quad \rightarrow \vec{n} \begin{Bmatrix} \frac{6}{7} \\ \frac{3}{7} \\ \frac{2}{7} \end{Bmatrix}$$

$$\{t\} = [\sigma]\{n\}$$

$$\begin{Bmatrix} t_x \\ t_y \\ t_z \end{Bmatrix} = \begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{Bmatrix} 6 \\ 3 \\ 2 \end{Bmatrix} \frac{1}{7} = \frac{1}{7} \begin{Bmatrix} 6 \\ -1 \\ 7 \end{Bmatrix} \text{ MPa}$$

$$\sigma = \vec{t}\cdot\vec{n} = \frac{1}{7}\langle 6 \; -1 \; 7\rangle \frac{1}{7}\begin{Bmatrix} 6 \\ 3 \\ 2 \end{Bmatrix} = \frac{1}{49}(36-3+14) = \frac{47}{49} \text{ MPa.}$$

$$\tau = \sqrt{\|\vec{t}\|^2 - \sigma^2} = \sqrt{\frac{1}{49}(6^2+1+7^2) - \left(\frac{47}{49}\right)^2}$$

$$= \frac{1}{7}\sqrt{86 - \frac{47^2}{49}} = \frac{1}{7}\sqrt{\frac{86\cdot 49 - 47^2}{49}}$$

$$\boxed{\tau = \sqrt{\frac{2005}{49}} \text{ MPa}}$$

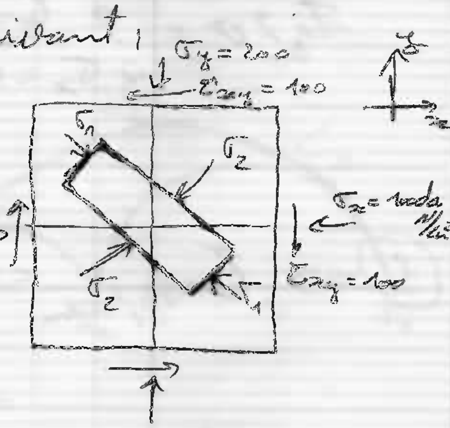
$$\boxed{t^2 = \tau^2 + \sigma^2}$$

Fin

TD3

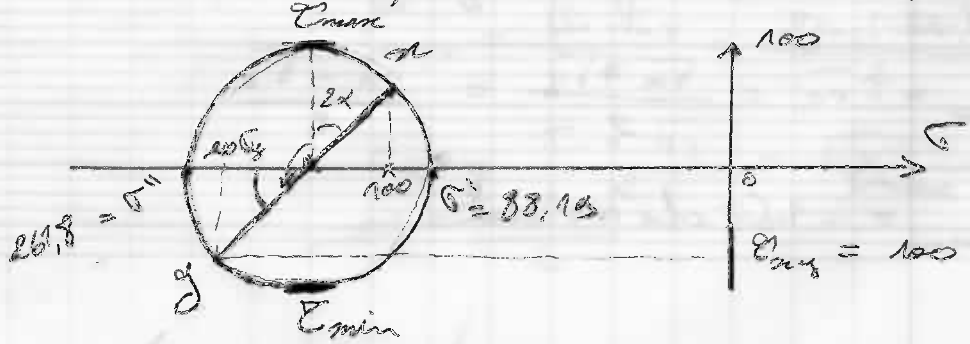
Exo 1

On donne l'état de c^nt plan suivant,



1 → Déterminer les c^nt principale et leur direction.

2 → Déterminer les c^nt τ_g ^ètte ~~normale~~ min et max et leur direction et la c^nt normale corespondent.



1 $\Rightarrow \sigma' = 0/x \qquad OO = \frac{\sigma_x + \sigma_y}{2}$

$$\sigma'' = \frac{180 - 20}{2} / y \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\left.\begin{matrix}\sigma' \\ \sigma''\end{matrix}\right\} = \frac{1}{2}(\sigma_x + \sigma_y) \mp \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma' = -38{,}19\ daN/cm^2 = \sigma_1$

$\sigma_1 > \sigma_2 > \sigma_3$

$\sigma'' = -261{,}8\ daN/cm^2 = \sigma_2$

$$tg\,2\theta = \frac{2\,\tau_{xy}}{\sigma_x - \sigma_y} \Rightarrow \boxed{\theta = -31{,}71 \pm 180°}$$

$= -31{,}71 - 90 \pm 180°$

2 $\Rightarrow \tau_{max/min} = \pm R = \frac{\sigma_1 - \sigma_2}{2} = \pm 111{,}8\ daN/cm^2$

$$= \pm \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^2}$$

$$\sigma_{\tau_{max}} = OO' = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_1 + \sigma_2}{2}$$

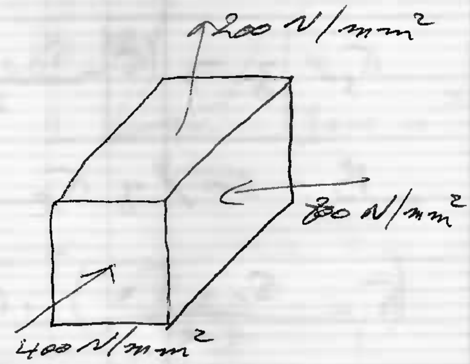
$\sigma_{\tau_{max}} = -150\ daN/cm^2$

$$tg\, 2\alpha = \frac{\frac{\sigma_x - \sigma_y}{2}}{\tau_{xy}} = 13^\circ$$

Pour avoire les C^{tte} maximale il faut tourne à l'angle de 13°

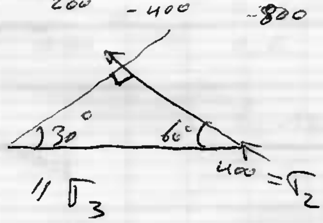
Exo 2 :

Soit l'état de σ suivants :

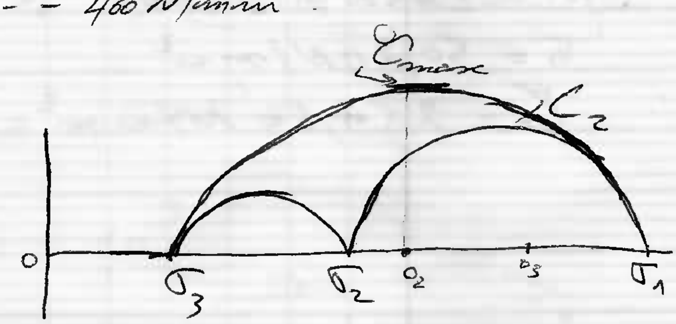


1) Déterminer τ_{max}.
2) Déterminer σ et τ pour le plan qui est // à $\sigma = 800\ N/mm^2$ et qui fait un angle de 30° avec $\sigma = -400\ N/mm^2$.

→ On à : $\underset{200}{\sigma_1} > \underset{-400}{\sigma_2} > \underset{-800}{\sigma_3}$.



$\beta = 60/\sigma_2$



$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{200-(-800)}{2} = 500\ N/mm^2$$

$$OO_2 = \frac{\sigma_1 + \sigma_3}{2} = \sigma_{\tau_{max}}$$

$$OO_2 = \frac{200+(-800)}{2} = -300$$

2) $\sigma \parallel \sigma_3 \Rightarrow$ il appartient au cercle.
$\Rightarrow \perp$ au plan z.

$$\left(\sigma - \frac{\sigma_1+\sigma_2}{2}\right)^2 + \tau^2 = (\sigma_1-\sigma_3)(\sigma_2-\sigma_3)\,n^2 + \left(\frac{\sigma_1-\sigma_2}{2}\right)^2$$

car ($\parallel z \Rightarrow n = 0$)

$$\sigma_3 \parallel z \rightarrow \left(\sigma - \frac{\sigma_1+\sigma_2}{2}\right)^2 + \tau^2 = \left(\frac{\sigma_1-\sigma_2}{2}\right)^2$$

$\beta = 60 / z\ \sigma_2$

$$\left(\sigma - \frac{\sigma_1+\sigma_3}{2}\right)^2 + \tau^2 = (\sigma_1-\sigma_2)(\sigma_3-\sigma_2)\,m^2 + \left(\frac{\sigma_1-\sigma_3}{2}\right)$$

$m = \cos\beta = \cos 60$

$\sigma = 50{,}29\ N/mm^2$

$\tau = 259{,}66\ N/mm^2$

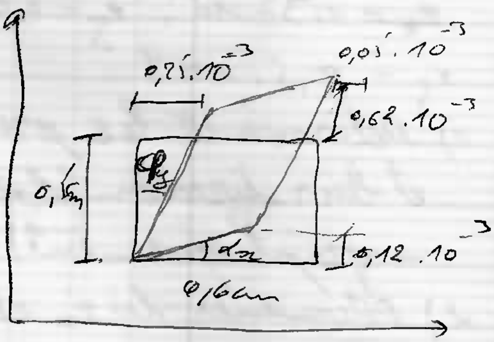
3/11/2011

TD N°1 : Etat de ε.

Exo1 :

Soit le domaine plan déformé selon la configuration de dessus

-> Déterminer le tenseur de déformation pour le plan (xoy) ?



Solution :

u, v ?

$u = x' - x \qquad V = y' - y$

$u = [-0,05 - (+0,25)] \cdot 10^{-3} \Rightarrow \boxed{u = -0,3 \cdot 10^{-3}\ mm}$

$V = 0,62 \cdot 10^{-3} - 0,12 \cdot 10^{-3} \Rightarrow \boxed{V = 0,5\ 10^{-3}\ mm}$

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{-0,3 \cdot 10^{-3}}{0,6} = 0,5 \cdot 10^{-3}$$

$$\varepsilon_y = \frac{\partial V}{\partial y} = \frac{0,5 \cdot 10^{-3}}{0,5} = 10^{-3}$$

$$\gamma_{xy} = \left(\frac{\partial u}{\partial y} + \frac{\partial V}{\partial x}\right) = \alpha_x + \alpha_y = \overbrace{\frac{0,25 \cdot 10^{-3}}{0,5}}^{\alpha_y} + \overbrace{\frac{0,12 \cdot 10^{-3}}{0,6}}^{\alpha_x}$$

$$\boxed{\gamma_{xy} = 0,716 \cdot 10^{-3}}$$

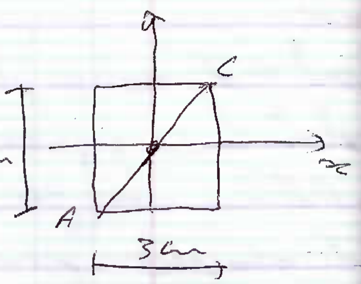
$$\varepsilon_{xy} = \frac{1}{2} \gamma_{xy}$$

Exo2:

Soit l'état de déformation ε de plan suivant

$\varepsilon_x = 1{,}37 \cdot 10^{-4}$, $\varepsilon_y = 0{,}125 \cdot 10^{-4}$

$\varepsilon_z = 2{,}5 \cdot 10^{-4}$.



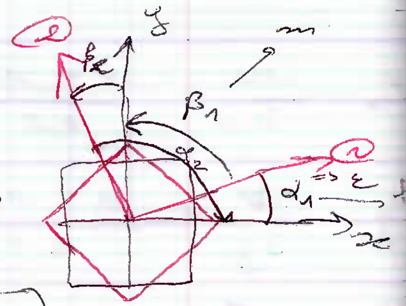
1) Déterminer les ε principale et leur direction (Analytiquement + graphiquement)?

2.) Déterminer les ε_x et γ_{xy} sur la facette AC ainsi que ~~Variant~~ variation ΔL_{AC}.

Solution:

1) → $\varepsilon_{1,2} = \dfrac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\dfrac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\dfrac{\gamma_{xy}}{2}\right)^2}$

$$\begin{cases} \varepsilon_1 = 2{,}14 \cdot 10^{-4} \\ \varepsilon_2 = 0{,}68 \cdot 10^{-4} \end{cases}$$



$\operatorname{tg} 2\alpha_1 = \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \dfrac{2{,}5 \cdot 10^{-4}}{(1{,}37 - 0{,}125)\, 10^{-4}} = 2{,}008$

$\boxed{\alpha_1 = 31{,}76^\circ}$

2) → $\varepsilon_{xA} = \varepsilon_x \cos^2\alpha + \varepsilon_y \sin^2\alpha + \gamma_{xy} \sin\alpha \cos\alpha$.

$\gamma_{xy} = 2(\varepsilon_y - \varepsilon_x) \sin\alpha \cos\alpha + \gamma_{xy}(\cos^2\alpha - \sin^2\alpha)$.

Avec : $\alpha = 45^\circ$.

$$\begin{cases} \varepsilon'_x = 1{,}99 \cdot 10^{-4} \\ \gamma_{xy} = -1{,}24 \cdot 10^{-4} \end{cases}$$

$$\varepsilon'_x = \frac{\Delta L_{AC}}{L_{AC}} \Rightarrow \Delta L_{AC} = L_{AC} \cdot \varepsilon'_x$$

$$= \sqrt{18} \cdot 1{,}99 \cdot 10^{-4}$$

$$\boxed{\Delta L_{AC} = 8{,}44 \cdot 10^{-4}\,\text{cm}}$$

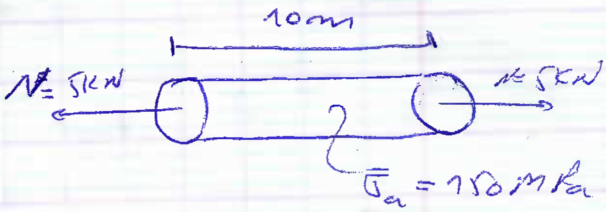
1 16/1/2012 Cha 5.

Exo1 :

Soit une barre de 10 m de longueur, elle doit transmettre un effort de traction de 5KN sans s'allongée plus de 3mm et sans que la (σ) ne depasse pas $\bar{\sigma}_a = 150 MPa$.

- Que doit etre le ϕ minimale de la barre ($E = 200000$ MPa).

Solution :



$\Delta l \leq 3mm$.

$\varepsilon l \leq 3mm$.

$\frac{\sigma}{E} l \leq 3mm$.

$$\sigma \leq \frac{\bar{\Delta l} \cdot E}{l} \qquad \bar{\Delta l} = 3mm$$

$$\sigma \leq \bar{\sigma} \Rightarrow \frac{N}{A} \leq \bar{\sigma} \Rightarrow A \geq \frac{N}{\sigma} \Rightarrow \boxed{r \geq \sqrt{\frac{N}{\pi \bar{\sigma}}}} \quad ①$$

$$\sigma \leq \frac{\bar{\Delta l}\, E}{l} \Rightarrow \frac{N}{\pi r^2} \leq \frac{\bar{\Delta l}\, E}{l} \Rightarrow \boxed{r \geq \sqrt{\frac{N l}{\bar{\Delta l}\, E \pi}}} \quad ②$$

$$\left.\begin{array}{l} ① \Rightarrow r \geq 3,26 \text{ cm} \\ ② \Rightarrow r \geq 5,15 \text{ cm} \end{array}\right\} \Rightarrow \boxed{r_{finale} \geq 5,15 \text{ cm}}$$

Exo 2:

Soit un etat de σ au pt $M(x,y,z)$

$\sigma_x = \sigma_y = \sigma_z = 0 \qquad \tau_{xy} = 0 \qquad \tau_{xz} = -Gay,$

$\tau_{yz} = +Gax.$

a : constante , G : module de glissement

- Determiner les composantes du vecteur deplacement (u, v, w) au pt M.

Solution :

$$0 = \varepsilon_x = \frac{\partial u}{\partial x} \qquad \gamma_{xy} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$0 = \varepsilon_y = \frac{\partial v}{\partial y} \qquad \gamma_{xz} = \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = -ay$$

$$0 = \varepsilon_z = \frac{\partial w}{\partial z} \qquad \gamma_{yz} = \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = ax$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] = 0$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] = 0$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] = 0$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0, \quad \gamma_{xz} = \frac{\tau_{xz}}{G} = -ay, \quad \gamma_{yz} = \frac{\tau_{yz}}{G} = ax$$

$$\varepsilon_x = 0 \Rightarrow \frac{\partial u}{\partial x} = 0 \Rightarrow u = u(y,z)$$

$$\varepsilon_y = 0 \Rightarrow \frac{\partial v}{\partial y} = 0 \Rightarrow v = v(x,z).$$

$$\varepsilon_z = 0 \Rightarrow \frac{\partial w}{\partial z} = 0 \Rightarrow w = w(x,y).$$

$$\gamma_{xy} = 0 \Rightarrow \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = 0 \Rightarrow \boxed{\frac{\partial u_0(y,z)}{\partial y} = -\frac{\partial v_0(x,z)}{\partial x}}$$ ③

$$\gamma_{yz} = \frac{\partial v_0(x,z)}{\partial z} + \frac{\partial w_0(x,y)}{\partial y} = ax \quad \text{— ① } /\partial x$$

$$\gamma_{xz} = \frac{\partial u_0(y,z)}{\partial z} + \frac{\partial w_0(x,y)}{\partial x} = -ay \quad \text{— ② } /\partial y$$

$$\frac{\partial}{\partial z}\left[\frac{\partial v_0(x,z)}{\partial x}\right] + \frac{\partial^2 w_0(x,y)}{\partial x \partial y} = a \quad \text{— ④}$$

$$\frac{\partial}{\partial z}\left[\frac{\partial u_0(y,z)}{\partial y}\right] + \frac{\partial^2 w_0(x,y)}{\partial x \partial y} = -a \quad \text{— ⑤}$$

⑤ - ④ → remplacé en ③

$$\frac{\partial}{\partial z}\left[\frac{\partial u_0(y,z)}{\partial y}\right] = -a \qquad \text{en } \int \text{ à } z$$

$$u_0(y,z) = -ayz + \int c_1(y)\,dy + c_2$$

$$\frac{\partial v_0(x,z)}{\partial x} = az - \underbrace{c_1(y)}_{=\text{cst ou } 0} \qquad \text{après ③.}$$

on pose ; $c_1(y) = c_1$

$$\begin{cases} v_0(x,z) = axz - c_1 x + C_2 \\ u_0(y,z) = -ayz + c_1 y + c_3 \end{cases}$$

d'après ①

$$\frac{\partial w_0(x,y)}{\partial y} = ax - \underbrace{\frac{\partial v_0(x,z)}{\partial z}}_{ax} = 0 \qquad w_0(x,y) = C_4(x)$$